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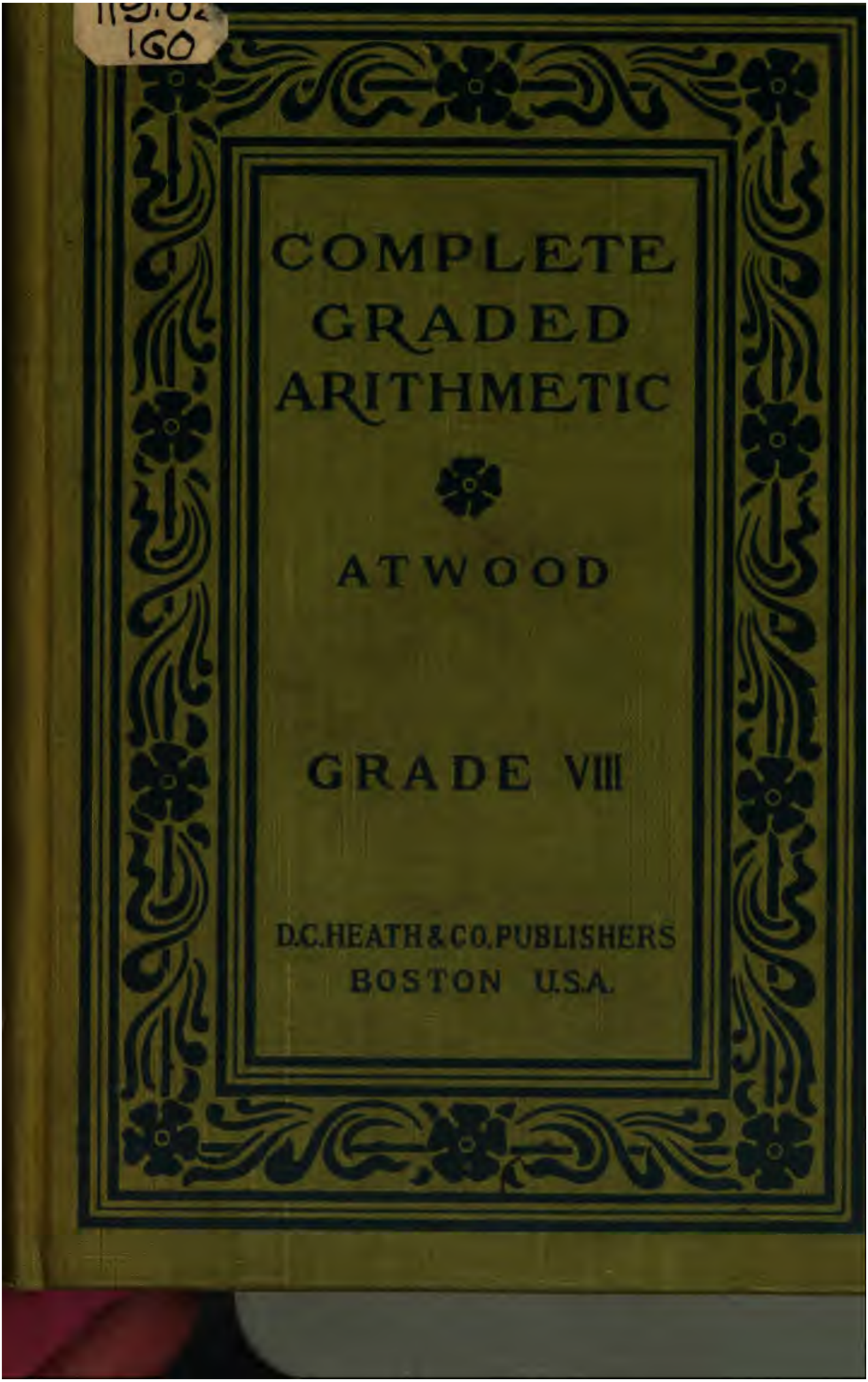

COMPLETE GRADED ARITHMETIC

ATWOOD

GRADE VIII

D.C. HEATH & CO. PUBLISHERS
BOSTON U.S.A.

Educ T 119.02.160
Grade 6

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COMPLETE

GRADED ARITHMETIC

EIGHTH GRADE

BY

GEORGE E. ATWOOD

BOSTON, U.S.A.
D. C. HEATH & CO., PUBLISHERS
1902

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PREFACE

THE work usually undertaken in Arithmetic in the last year of the grammar school course is a general review of the subjects studied in the preceding grades and the study of those topics that have not been presented before. This book has been made to meet these requirements. Beginning with divisibility of numbers, the book contains a thorough review of all the subjects presented in the other books of the series. Then follows the study of partnership, ratio and proportion, involution, evolution, longitude and time, metric system, mensuration, and the elements of Algebra.

The plan of the book differs materially from the other books of the series. The purpose for which it is made makes the topical arrangement necessary. Accompanying each topic will be found a sufficiently large number of examples and problems to illustrate the principles involved. The book also differs from the others in that it contains no oral work except on those pages devoted to Algebra. In view of the abundance of oral exercises contained in the preceding books, it is not so important that such exercises should be continued through this grade. But if teachers wish to continue the mental arithmetic in eighth grade classes, they will find it quite convenient to make use of the oral pages found in the fifth, sixth, and seventh grade books.

A little more of the formal work of Algebra is introduced into this book. The work begins here, as it does in the seventh grade book, with exercises that are intended to develop the idea of representing numbers by letters, the meaning of the equation, and its application to the solution of problems. Following these pages will be found the treatment of the fundamental processes of addition, subtraction, multiplication, and division. Under each of these heads are enough examples to develop a good degree of skill in these algebraic processes. These pages also contain several exercises in finding the numerical value of algebraic expressions, which are given for the purpose of making clear the law of the algebraic notation and the signification of the coefficient and exponent. Here is about as much of Algebra as most teachers will think best to attempt in the grammar school course. Pupils who do this work intelligently will be well prepared to take up the academic course in Algebra.

In the preparation of this book the author has received valuable assistance from F. E. Spaulding, Superintendent of Schools, Passaic, N. J.; J. C. Lyford, Principal of Winslow Street School, Worcester, Mass.; Charles L. Van Cleve, Superintendent of Schools, Troy, Ohio; and W. H. Baker, Instructor in Mathematics, State Normal School, San Jose, Cal., for which he desires to make grateful acknowledgment.

G. E. A.

COMPLETE GRADED ARITHMETIC

EIGHTH GRADE

An **Exact Divisor** of a number is a number that is exactly contained in that number.

A **Prime Number** is a number that has no exact divisors except itself and 1; as, 11, 17, 23, 29.

A **Composite Number** is a number that has exact divisors besides itself and 1; as, 15, 18, 45, 63.

An **Even Number** is a number that is exactly divisible by 2. If even, the right-hand figure is even or 0.

An **Odd Number** is a number that is not exactly divisible by 2; as, 7, 11, 19, 33.

Any number is exactly divisible:

By 3, if the sum of its digits is divisible by 3; as, 738, 825, 804, 498, 888.

By 4, if the number expressed by its two right-hand digits is divisible by 4; as, 728, 936, 760, 544.

By 5, if its right-hand digit is 5 or 0; as, 475, 730.

By 6, if it is even and is divisible by 3; as, 534, 738.

By 8, if the number expressed by its 3 right-hand digits is divisible by 8; as, 24,720, 61,000, 23,048, 45,000.

By 9, if the sum of its digits is divisible by 9; as, 603, 828, 1008, 21,312, 73,305.

FACTORING

The **Factors** of a number are the exact divisors of it. All exact divisors of a number are factors of it.

Prime Factors are factors that are prime numbers.

The **Common Prime Factors** of numbers are the numbers that are exact divisors of each of them.

Numbers are *prime to each other* when they have no common factor.

TO FIND THE PRIME FACTORS

$$\begin{array}{r|r}
2 & 210 \\ \cline{2-2}
3 & 105 \\ \cline{2-2}
5 & 35 \\ \cline{2-2}
 & 7
\end{array}
\qquad
\begin{array}{r|r}
2 & 140 \\ \cline{2-2}
2 & 70 \\ \cline{2-2}
5 & 35 \\ \cline{2-2}
 & 7
\end{array}
\qquad
\begin{array}{r|r}
2 & 126 \\ \cline{2-2}
3 & 63 \\ \cline{2-2}
3 & 21 \\ \cline{2-2}
 & 7
\end{array}
\qquad
\begin{array}{r|r}
2 & 770 \\ \cline{2-2}
5 & 385 \\ \cline{2-2}
7 & 77 \\ \cline{2-2}
 & 11
\end{array}$$

2, 3, 5, 7   2, 2, 5, 7   2, 3, 3, 7   2, 5, 7, 11

The *common prime factors* of the numbers 210, 140, 126, 770, are 2 and 7, for they are factors of each of them.

RULE. — *Divide the number by any prime number that is an exact divisor of it; then divide the quotient, if composite, by any prime number that is an exact divisor of it; and so on, until the quotient is a prime number. The several divisors and the last quotient are the prime factors.*

EXAMPLES IN FACTORING

1. 272.
2. 330.
3. 1625.
4. 1872.
5. 693.
6. 484.
7. 2310.
8. 1312.
9. 441.
10. 585.
11. 1197.
12. 3025.

CANCELLATION

PRINCIPLE. — *Dividing both dividend and divisor by the same number does not change the quotient.*

$$\begin{array}{r}12\underline{)48}\\4\end{array} \qquad \begin{array}{r}6\underline{)24}\\4\end{array} \qquad \begin{array}{r}4\underline{)16}\\4\end{array} \qquad \begin{array}{r}3\underline{)12}\\4\end{array}$$

By applying this principle in dividing the product of two or more numbers by a single number or the product of two or more numbers, the work is much shortened.

$$(28 \times 5) \div 35 \qquad \overline{15 \times 12} \div 9 \qquad (28 \times 27) \div 12$$

$$\frac{\overset{4}{\cancel{28}} \times \cancel{5}}{\underset{\cancel{7}}{\cancel{35}}} = 4 \qquad \frac{\overset{5}{\cancel{15}} \times \overset{4}{\cancel{12}}}{\underset{\cancel{3}}{\cancel{9}}} = 20 \qquad \frac{\overset{7}{\cancel{28}} \times \overset{9}{\cancel{27}}}{\underset{\cancel{3}}{\cancel{12}}} = 63$$

Cancellation is the process of shortening division by rejecting equal factors from dividend and divisor.

1. (45 × 68) ÷ (17 × 15).
2. (90 × 55) ÷ (27 × 25).
3. (49 × 39) ÷ (26 × 14).
4. (85 × 64) ÷ (42 × 34).
5. If 24 acres of land cost $258, what will 90 acres cost at the same price?
6. How many days will 150 bushels of oats last 8 horses, if each horse eats 10 quarts a day?
7. How many dozen eggs, at 18 cents a dozen, will pay for 450 pounds of sugar at 5 cents a pound?

LEAST COMMON MULTIPLE

A **Multiple** of a number is a number that is exactly divisible by that number. 35, 60, and 85 are multiples of 5. 28, 42, 49, 56, and 63 are multiples of 7.

A **Common Multiple** of numbers is a number that is exactly divisible by each of them. 70 is a common multiple of 5 and 7. 36 and 60 are common multiples of 3 and 4.

The **Least Common Multiple** of numbers is the smallest number that is exactly divisible by each of them. 90 is the least common multiple of 6, 9, and 10.

From the definitions of multiple and least common multiple these principles are evident.

PRINCIPLES. — *A multiple of a number contains all the prime factors of that number.*

The least common multiple of several numbers contains only the prime factors of all the numbers.

2	30	40	42	60	70	105
5	15	20	21	30	35	105
7	3	4	21	6	7	21
3	3	4	3	6	1	3
2	1	4	1	2	1	1
	1	2	1	1	1	1

Dividing these numbers by any prime number that is an exact divisor of two or more of them and continuing the process until the quotients are prime to each other, the several divisors and final quotients represent all the prime factors of the numbers. Hence their product, 840, is the least common multiple of the numbers.

EXAMPLES IN LEAST COMMON MULTIPLE

1. 10, 64, 96.
2. 315, 420.
3. 48, 60, 180.
4. 50, 70, 95.
5. 560, 616.
6. 22, 55, 154.
7. 36, 40, 48.
8. 432, 936.
9. 35, 42, 126.
10. 63, 81, 126, 135.
11. 16, 20, 48, 15, 60.
12. 12, 24, 108, 192.
13. 20, 45, 35, 28, 40.
14. 66, 99, 135, 168.
15. 18, 27, 21, 36, 63.
16. 32, 64, 112, 160.
17. 12, 15, 20, 28, 35.

GREATEST COMMON DIVISOR

A **Common Divisor** of numbers is a number that is an exact divisor of each of them.

The **Greatest Common Divisor** of numbers is the largest number that is an exact divisor of each of them.

Principle. — *The greatest common divisor of numbers is the product of their common factors.*

GREATEST COMMON DIVISOR BY FACTORING

$$\begin{array}{r|r} 2 & 42 \\ \cline{2-2} 3 & 21 \\ \cline{2-2} & 7 \end{array} \qquad \begin{array}{r|r} 2 & 70 \\ \cline{2-2} 5 & 35 \\ \cline{2-2} & 7 \end{array} \qquad \begin{array}{r|r} 2 & 154 \\ \cline{2-2} 7 & 77 \\ \cline{2-2} & 11 \end{array} \qquad \begin{array}{r|r} 2 & 182 \\ \cline{2-2} 7 & 91 \\ \cline{2-2} & 13 \end{array}$$

$$2,\ 3,\ 7 \qquad 2,\ 5,\ 7 \qquad 2,\ 7,\ 11 \qquad 2,\ 7,\ 13$$

The numbers, 42, 70, 154, 182, have the common factors, 7 and 2; hence 7 × 2, or 14, is their G. C. D.

1. Find the greatest common divisor of 169 and 195. Of 187 and 418. Of 433 and 977. Of 521 and 971.

2. Find the greatest common divisor of 120, 210, and 345. Of 352, 672, and 992. Of 316, 360, and 432.

GREATEST COMMON DIVISOR BY DIVISION

When the numbers cannot be factored readily by inspection, the greatest common divisor is found by the method of continued division. This method of finding the greatest common divisor is based on these principles:

PRINCIPLES. — *A divisor of any number is a divisor of any multiple of that number.*

A common divisor of two numbers is a divisor of their sum and also of their difference.

Find G. C. D. of 1134 and 1386. Of 1815 and 3465.

1134)1386(1
1134
252)1134(4
1008
126)252(2
252

126 is the G. C. D.

1815)3465(1
1815
1650)1815(1
1650
165)1650(10
1650

165 is the G. C. D.

RULE. — *Divide the greater number by the less; divide the last divisor by the last remainder, until there is no remainder. The last divisor is the greatest common divisor.*

EXAMPLES

1. 768, 5184.
2. 2041, 8476.
3. 1197, 2375.
4. 630, 1386.
5. 3768, 3596.
6. 1479, 4959.

1. Find the smallest number that is exactly divisible by 338, 364, and 448.

2. How many times is the greatest common divisor of 48, 60, and 180 contained in their least common multiple?

3. How long is the shortest piece of wire that can be cut without waste into pieces 16, 18, or 22 ft. long?

4. Find the smallest sum that could be exactly expended in purchasing cows at $24, $27, $32, $36, or $56 apiece.

5. A merchant wishes to buy hats at $3, $4, $5, and $6 apiece and to expend the same sum for each kind. What is the smallest sum that he can invest in the purchase?

6. A grocer wishes to buy tea at 56, 63, and 72 cents a pound and to expend the same sum for each kind. Find the least number of pounds he can purchase.

7. A man has two farms containing 78 acres and 324 acres respectively. Into how many equal fields can he divide them and have the fields as large as possible?

8. A man has four lots which are 120, 420, 600, and 900 ft. wide respectively. What is the least number of lots of equal width into which he can divide them?

9. The sides of a garden are 168, 182, 252, and 280 ft. respectively. Using as long boards as possible, how many will it take for a fence 4 boards high?

10. A farmer has 260 bushels of rye, 384 bushels of oats, and 416 bushels of wheat. Find the least number of bags of equal size in which he can take the grain to market.

11. A merchant has three kinds of tea; 60 pounds of the first, 75 pounds of the second, and 100 pounds of the third. Find the least number of equal packages into which he can put it without mixing the different kinds of tea.

FRACTIONS

A **Fraction** of anything is one or more of the equal parts into which it is divided.

If anything is divided into 3 equal parts, the parts are thirds; if anything is divided into 4 equal parts, the parts are fourths; if anything is divided into 5 equal parts, the parts are fifths; and so on.

To express a fraction of anything, it is necessary to use one number to denote how many of the equal parts are thought of, and another number or word to denote into how many equal parts it is divided.

The **Numerator** is the number that denotes how many of the equal parts are thought of.

The **Denominator** is the number that denotes the name of the equal parts into which anything is divided.

PRINCIPLES. — *Dividing the numerator by any number divides the fraction by that number.*

When we divide the numerator, we divide the number of equal parts while the magnitude of the parts remains the same.

Dividing the denominator by any number multiplies the fraction by that number.

When we divide the denominator, we increase the magnitude of the equal parts while the number of parts remains the same.

Dividing the numerator and denominator by the same number does not change the value of the fraction.

Since dividing the numerator divides the fraction and dividing the denominator multiplies the fraction, when we divide both by the same number, we first divide the fraction and then multiply it by the same number, and the value of the fraction is not changed.

Reduction of Fractions is the process of changing their form without changing their value.

REDUCING FRACTIONS TO LOWEST TERMS

A fraction is reduced to its lowest terms when the numerator and denominator are prime to each other.

$\frac{18}{63} = \frac{2}{7}$ $\frac{24}{60} = \frac{2}{5}$ $\frac{56}{64} = \frac{7}{8}$ $\frac{24}{56} = \frac{3}{7}$

RULE. — *Divide the numerator and denominator by their greatest common factor.*

EXAMPLES

1. $\frac{38}{57}$, $\frac{26}{65}$, $\frac{24}{84}$. 2. $\frac{63}{72}$, $\frac{42}{63}$, $\frac{42}{70}$. 3. $\frac{48}{64}$, $\frac{40}{72}$, $\frac{46}{69}$.

4. $\frac{64}{75}$, $\frac{48}{72}$, $\frac{51}{68}$. 5. $\frac{56}{70}$, $\frac{36}{84}$, $\frac{54}{72}$. 6. $\frac{52}{78}$, $\frac{36}{63}$, $\frac{30}{75}$.

7. $\frac{45}{75}$, $\frac{26}{91}$, $\frac{54}{81}$. 8. $\frac{32}{72}$, $\frac{60}{75}$, $\frac{56}{72}$. 9. $\frac{65}{78}$, $\frac{32}{80}$, $\frac{56}{70}$.

A **Proper Fraction** is a fraction whose numerator is less than the denominator.

An **Improper Fraction** is a fraction whose numerator equals or exceeds the denominator.

A **Mixed Number** is a whole number and a fraction written together.

REDUCING FRACTIONS TO WHOLE OR MIXED NUMBERS

$\frac{26}{3} = 8\frac{2}{3}$ $\frac{27}{4} = 6\frac{3}{4}$ $\frac{48}{5} = 9\frac{3}{5}$ $\frac{45}{8} = 5\frac{5}{8}$

RULE. — *Divide the numerator by the denominator.*

EXAMPLES

1. $\frac{54}{4}$, $\frac{67}{5}$, $\frac{79}{6}$. 2. $\frac{69}{5}$, $\frac{57}{4}$, $\frac{80}{6}$. 3. $\frac{81}{6}$, $\frac{73}{5}$, $\frac{59}{4}$.

4. $\frac{68}{5}$, $\frac{55}{4}$, $\frac{82}{6}$. 5. $\frac{62}{4}$, $\frac{74}{5}$, $\frac{89}{7}$. 6. $\frac{58}{4}$, $\frac{85}{6}$, $\frac{77}{5}$.

7. $\frac{71}{5}$, $\frac{66}{4}$, $\frac{93}{7}$. 8. $\frac{61}{4}$, $\frac{87}{6}$, $\frac{82}{5}$. 9. $\frac{70}{4}$, $\frac{72}{5}$, $\frac{92}{7}$.

REDUCING MIXED NUMBERS TO IMPROPER FRACTIONS

$4\frac{4}{5} = \frac{24}{5}$ $3\frac{5}{6} = \frac{23}{6}$ $7\frac{4}{7} = \frac{53}{7}$ $4\frac{7}{8} = \frac{39}{8}$

RULE. — *Multiply the denominator of the fraction by the whole number, to the product add the numerator of the fraction, and write the sum over the denominator.*

EXAMPLES

1. $14\frac{5}{7}$, $17\frac{3}{8}$, $19\frac{7}{9}$. **2.** $15\frac{8}{9}$, $18\frac{4}{9}$, $20\frac{7}{8}$. **3.** $16\frac{5}{6}$, $13\frac{5}{8}$, $17\frac{6}{7}$.

4. $19\frac{4}{5}$, $20\frac{4}{7}$, $16\frac{8}{9}$. **5.** $14\frac{7}{9}$, $13\frac{4}{9}$, $15\frac{7}{8}$. **6.** $18\frac{5}{8}$, $19\frac{5}{9}$, $17\frac{4}{7}$.

REDUCING FRACTIONS TO A COMMON DENOMINATOR

PRINCIPLES. — *Multiplying the numerator by any number multiplies the fraction by that number.*

When we multiply the numerator, we multiply the number of equal parts while the magnitude of the parts remains the same.

Multiplying the denominator by any number divides the fraction by that number.

When we multiply the denominator, we diminish the magnitude of the equal parts while the number of parts remains the same.

Multiplying numerator and denominator by the same number does not change the value of the fraction.

Since multiplying the numerator multiplies the fraction and multiplying the denominator divides the fraction, when we multiply both by the same number, we first multiply the fraction and then divide it by the same number, and the value of the fraction is not changed.

$\frac{1}{2}=\frac{6}{12}$ $\frac{2}{3}=\frac{8}{12}$ $\frac{2}{3}=\frac{12}{18}$ $\frac{2}{3}=\frac{16}{24}$

$\frac{1}{3}=\frac{4}{12}$ $\frac{3}{4}=\frac{9}{12}$ $\frac{5}{6}=\frac{15}{18}$ $\frac{5}{6}=\frac{20}{24}$

$\frac{1}{4}=\frac{3}{12}$ $\frac{5}{6}=\frac{10}{12}$ $\frac{7}{9}=\frac{14}{18}$ $\frac{7}{8}=\frac{21}{24}$

RULE. — *Find the least common multiple of the denominators. Divide the least common multiple by the denominator of each fraction, and multiply numerator and denominator of the fraction by the quotient.*

EXAMPLES

1. $\frac{5}{6}, \frac{7}{9}, \frac{7}{12}$. **2.** $\frac{3}{4}, \frac{7}{12}, \frac{17}{20}$. **3.** $\frac{6}{7}, \frac{11}{28}, \frac{13}{35}$.

4. $\frac{5}{6}, \frac{4}{7}, \frac{8}{21}$. **5.** $\frac{3}{5}, \frac{11}{20}, \frac{17}{30}$. **6.** $\frac{5}{9}, \frac{17}{24}, \frac{11}{36}$.

7. $\frac{2}{3}, \frac{7}{8}, \frac{11}{12}$. **8.** $\frac{4}{5}, \frac{11}{12}, \frac{9}{20}$. **9.** $\frac{5}{7}, \frac{11}{14}, \frac{17}{35}$.

ADDITION OF FRACTIONS

Fractions can be added and subtracted only when they are like equal parts, and they express like equal parts only when they have a common denominator.

$$\begin{array}{r} 2\frac{1}{2}=2\frac{6}{12} \\ 3\frac{1}{3}=3\frac{4}{12} \\ 6\frac{1}{4}=6\frac{3}{12} \\ \hline 12\frac{1}{12} \end{array} \qquad \begin{array}{r} 4\frac{1}{2}=4\frac{9}{18} \\ 5\frac{2}{3}=5\frac{12}{18} \\ 6\frac{7}{9}=6\frac{14}{18} \\ \hline 16\frac{17}{18} \end{array} \qquad \begin{array}{r} 5\frac{2}{3}=5\frac{16}{24} \\ 4\frac{3}{4}=4\frac{18}{24} \\ 8\frac{5}{8}=8\frac{15}{24} \\ \hline 19\frac{1}{24} \end{array}$$

EXAMPLES

1. $4\frac{2}{3}+7\frac{3}{4}+9\frac{1}{2}$. **2.** $7\frac{3}{4}+5\frac{3}{5}$. **3.** $5\frac{4}{5}+6\frac{1}{2}+3\frac{3}{4}$.

4. $8\frac{3}{4}+7\frac{5}{6}+4\frac{2}{3}$. **5.** $9\frac{5}{8}+4\frac{2}{3}$. **6.** $9\frac{3}{5}+8\frac{7}{8}+6\frac{3}{4}$.

7. $7\frac{1}{2}+9\frac{2}{3}+8\frac{3}{4}$. **8.** $6\frac{5}{6}+8\frac{3}{4}$. **9.** $7\frac{1}{2}+5\frac{5}{6}+4\frac{4}{5}$.

SUBTRACTION OF FRACTIONS

$$\begin{array}{cccc}
8\frac{5}{6} = 8\frac{5}{6} & 9\frac{1}{3} = 9\frac{4}{12} & 8\frac{2}{3} = 8\frac{10}{15} & 9\frac{3}{8} = 9\frac{9}{24} \\
2\frac{1}{2} = 2\frac{3}{6} & 2\frac{3}{4} = 2\frac{9}{12} & 4\frac{4}{5} = 4\frac{12}{15} & 3\frac{2}{3} = 3\frac{16}{24} \\
\hline
6\frac{1}{3} & 6\frac{7}{12} & 3\frac{13}{15} & 5\frac{17}{24}
\end{array}$$

EXAMPLES

1. $9\frac{1}{2} - 4\frac{5}{7}$. **2.** $8\frac{1}{4} - 3\frac{4}{5}$. **3.** $7\frac{3}{4} - 2\frac{1}{3}$.

4. $9\frac{1}{3} - 4\frac{5}{8}$. **5.** $9\frac{1}{2} - 4\frac{5}{9}$. **6.** $8\frac{3}{5} - 3\frac{1}{4}$.

7. $6\frac{1}{2} - 2\frac{2}{3}$. **8.** $7\frac{1}{3} - 3\frac{3}{4}$. **9.** $8\frac{2}{5} - 2\frac{2}{3}$.

10. $5\frac{2}{3} - 2\frac{5}{6} + 5\frac{3}{8}$. **11.** $9\frac{1}{6} - 4\frac{3}{4}$. **12.** $8\frac{7}{8} - 3\frac{3}{4} + 6\frac{4}{5}$.

13. $8\frac{1}{6} - 4\frac{3}{4} + 7\frac{1}{2}$. **14.** $8\frac{1}{9} - 4\frac{5}{6}$. **15.** $9\frac{5}{6} - 5\frac{2}{3} + 4\frac{3}{4}$.

MULTIPLICATION OF FRACTIONS

Multiplying the numerator multiplies the number of equal parts while the magnitude of the parts remains the same, and dividing the denominator increases the magnitude of the equal parts while the number of parts remains the same. Hence the following principle:

A fraction is multiplied by an integer by multiplying the numerator or dividing the denominator.

$\frac{2}{5} \times 2 = \frac{4}{5}$. $\frac{4}{15} \times 3 = \frac{4}{5}$. $\frac{2}{3} \times 9 = 6$. $\frac{7}{9} \times 3 = 2\frac{1}{3}$.

EXAMPLES

1. $\frac{3}{5} \times 7$. **2.** $\frac{5}{12} \times 4$. **3.** $\frac{7}{9} \times 3$. **4.** $\frac{11}{15} \times 5$.

5. $\frac{7}{9} \times 6$. **6.** $\frac{7}{15} \times 6$. **7.** $\frac{3}{4} \times 7$. **8.** $\frac{17}{20} \times 8$.

9. A merchant sold $\frac{3}{4}$ of a yard of ribbon to each of 18 ladies. How many yards did he sell to all?

$$\begin{array}{r} 12\frac{2}{3} \\ 4 \\ \hline 50\frac{2}{3} \end{array} \qquad \begin{array}{r} 7\frac{2}{3} \\ 9 \\ \hline 69 \end{array} \qquad \begin{array}{r} 4\frac{3}{4} \\ 5 \\ \hline 23\frac{3}{4} \end{array} \qquad \begin{array}{r} 6\frac{3}{4} \\ 8 \\ \hline 54 \end{array} \qquad \begin{array}{r} 4\frac{5}{6} \\ 3 \\ \hline 14\frac{1}{2} \end{array}$$

EXAMPLES

1. $7\frac{2}{3} \times 4$. **2.** $8\frac{3}{4} \times 8$. **3.** $5\frac{2}{5} \times 7$.

4. $9\frac{3}{8} \times 9$. **5.** $6\frac{2}{7} \times 6$. **6.** $7\frac{5}{8} \times 8$.

7. $8\frac{5}{6} \times 3$. **8.** $7\frac{7}{8} \times 4$. **9.** $9\frac{8}{9} \times 7$.

12 × $\frac{3}{4}$ means, find $\frac{3}{4}$ of 12.

3 fourths of a number is 3 times 1 fourth of the number or 1 fourth of 3 times the number. In either case the product of the whole number and numerator is divided by the denominator. Hence cancellation may be applied.

$12 \times \frac{3}{4} = 9$. $5 \times \frac{2}{3} = 3\frac{1}{3}$. $4 \times \frac{3}{8} = 1\frac{1}{2}$. $14 \times \frac{5}{6} = 11\frac{2}{3}$.

EXAMPLES

1. $54 \times \frac{5}{8}$. **2.** $45 \times \frac{5}{6}$. **3.** $33 \times \frac{7}{9}$. **4.** $44 \times \frac{3}{8}$.

5. $34 \times \frac{5}{6}$. **6.** $45 \times \frac{3}{4}$. **7.** $28 \times \frac{2}{3}$. **8.** $38 \times \frac{4}{5}$.

9. $46 \times \frac{7}{8}$. **10.** $28 \times \frac{6}{7}$. **11.** $42 \times \frac{3}{5}$. **12.** $75 \times \frac{5}{6}$.

$$\begin{array}{r} 12 \\ 5\frac{2}{3} \\ \hline 68 \end{array} \qquad \begin{array}{r} 6 \\ 7\frac{3}{4} \\ \hline 46\frac{1}{2} \end{array} \qquad \begin{array}{r} 9 \\ 4\frac{4}{5} \\ \hline 43\frac{1}{5} \end{array} \qquad \begin{array}{r} 8 \\ 8\frac{3}{4} \\ \hline 70 \end{array} \qquad \begin{array}{r} 10 \\ 7\frac{3}{7} \\ \hline 74\frac{2}{7} \end{array}$$

1. $38 \times 6\frac{5}{8}$. **2.** $33 \times 8\frac{5}{6}$. **3.** $51 \times 5\frac{7}{9}$.

4. $64 \times 9\frac{3}{4}$. **5.** $53 \times 7\frac{4}{5}$. **6.** $56 \times 6\frac{3}{8}$.

7. $56 \times 5\frac{6}{7}$. **8.** $75 \times 9\frac{5}{6}$. **9.** $46 \times 8\frac{7}{8}$.

$\frac{4}{5} \times \frac{2}{3}$ means, find $\frac{2}{3}$ of $\frac{4}{5}$.

Since $\frac{1}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$, $\frac{2}{3}$ of $\frac{4}{5}$ is 2 times $\frac{4}{15}$, or $\frac{8}{15}$. The fractional part of $\frac{4}{5}$ is found by dividing the product of the numerators by the product of the denominators, in which process cancellation may be applied when there are common factors in numerators and denominators.

$\frac{3}{8} \times \frac{2}{3} = \frac{1}{4}$. $\frac{1}{8} \times \frac{1}{3} = \frac{1}{24}$. $\frac{4}{9} \times \frac{3}{4} = \frac{1}{3}$. $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$.

EXAMPLES

1. $\frac{4}{9} \times \frac{3}{8}$. **2.** $3\frac{3}{4} \times 8\frac{1}{3}$. **3.** $\frac{18}{35} \times \frac{7}{9}$. **4.** $4\frac{1}{5} \times 2\frac{6}{7}$.
5. $\frac{8}{9} \times \frac{3}{4}$. **6.** $5\frac{4}{9} \times 1\frac{5}{7}$. **7.** $\frac{25}{32} \times \frac{4}{5}$. **8.** $5\frac{5}{8} \times 2\frac{2}{5}$.
9. $\frac{5}{6} \times \frac{3}{5}$. **10.** $8\frac{2}{5} \times 5\frac{5}{6}$. **11.** $\frac{35}{36} \times \frac{4}{7}$. **12.** $5\frac{5}{6} \times 6\frac{3}{7}$.

DIVISION OF FRACTIONS

$\frac{3}{4} \div 3$ means, find $\frac{1}{3}$ of $\frac{3}{4}$.

Since $\frac{3}{4} \div 3$ means $\frac{1}{3}$ of $\frac{3}{4}$, it may be written $\frac{3}{4} \times \frac{1}{3}$.

$\frac{3}{4} \div 3 = \frac{3}{4} \times \frac{1}{3}$ $\frac{2}{3} \div 4 = \frac{2}{3} \times \frac{1}{4}$ $\frac{1}{3} \div 3 = \frac{1}{3} \times \frac{1}{3}$

EXAMPLES

1. $\frac{6}{7} \div 3$. **2.** $\frac{10}{11} \div 5$. **3.** $\frac{3}{5} \div 6$. **4.** $\frac{21}{25} \div 7$.
5. $\frac{8}{3} \div 4$. **6.** $\frac{15}{17} \div 9$. **7.** $\frac{4}{5} \div 8$. **8.** $\frac{42}{11} \div 6$.
9. $\frac{9}{4} \div 6$. **10.** $\frac{28}{13} \div 8$. **11.** $\frac{8}{9} \div 6$. **12.** $\frac{25}{14} \div 5$.

13. A man bought a farm for $4650, which was $\frac{3}{7}$ of his money. How much did he have left?

14. A man sold a house for $6250 and lost $\frac{1}{6}$ as much as it cost. Find the cost and the loss.

In dividing a mixed number by a small whole number, do not reduce the mixed number to an improper fraction.

$$\begin{array}{r}3\overline{)146\frac{1}{4}}\\ 48\frac{3}{4}\end{array}\qquad\begin{array}{r}4\overline{)135\frac{1}{5}}\\ 33\frac{4}{5}\end{array}\qquad\begin{array}{r}5\overline{)142\frac{2}{3}}\\ 28\frac{8}{15}\end{array}\qquad\begin{array}{r}3\overline{)206\frac{2}{3}}\\ 68\frac{8}{9}\end{array}$$

EXAMPLES

1. $136\frac{1}{2} \div 3$. **2.** $257\frac{2}{3} \div 4$. **3.** $287\frac{2}{9} \div 5$.

4. $173\frac{3}{5} \div 4$. **5.** $218\frac{4}{7} \div 6$. **6.** $234\frac{2}{3} \div 8$.

7. $394\frac{5}{8} \div 7$. **8.** $273\frac{7}{9} \div 8$. **9.** $245\frac{5}{7} \div 9$.

$4 \div \frac{2}{3}$ means, find how many $\frac{2}{3}$ in 4.

To find how many 3 in. in 4 ft., we change 4 ft. to inches and divide the number of inches by 3. To find how many 8 cents in $6, we change $6 to cents and divide the number of cents by 8. To find how many 2 thirds in 4, we change 4 to thirds and divide the number of thirds by 2.

$4 \div \frac{2}{3} = 12 \div 2$ $\frac{2}{3} \div \frac{2}{9} = 6 \div 2$ $9 \div 1\frac{2}{3} = 27 \div 5$

$7 \div \frac{3}{4} = 28 \div 3$ $\frac{3}{4} \div \frac{2}{3} = 9 \div 8$ $7 \div 1\frac{3}{4} = 28 \div 7$

Each result is obtained by changing dividend and divisor to like fractional parts and dividing the first numerator by the second.

In all cases the result may be obtained by inverting the divisor and proceeding as in multiplication.

EXAMPLES

1. $9\frac{3}{8} \div 3\frac{3}{4}$. **2.** $4\frac{3}{4} \div 2\frac{1}{2}$. **3.** $6\frac{2}{3} \div 2\frac{7}{9}$.

4. $6\frac{2}{7} \div 2\frac{3}{4}$. **5.** $9\frac{1}{3} \div 2\frac{1}{3}$. **6.** $5\frac{5}{6} \div 2\frac{1}{3}$.

7. $8\frac{2}{3} \div 5\frac{7}{9}$. **8.** $9\frac{3}{5} \div 2\frac{3}{7}$. **9.** $4\frac{1}{6} \div 3\frac{1}{3}$.

1. A farmer sold $\frac{4}{7}$ of his oats and had 495 bushels left. How many bushels did he sell?

2. $5\frac{2}{3} - 2\frac{5}{6} + 5\frac{3}{8}$. **3.** $7\frac{5}{9} + 8\frac{5}{6}$. **4.** $3\frac{3}{4} + 5\frac{4}{5} - 3\frac{5}{6}$.

5. If 2 ninths of a piece of land is worth $656, how much is $\frac{4}{7}$ of the remainder worth?

6. $8\frac{1}{2} + 7\frac{2}{3} - 5\frac{3}{4}$. **7.** $8\frac{2}{3} - 4\frac{4}{5}$. **8.** $4\frac{5}{6} - 2\frac{8}{9} + 6\frac{2}{3}$.

9. A farmer sold sheep at $$3\frac{3}{4}$ a head, receiving $150 for them. How many did he sell?

10. $5\frac{3}{5} \div 1\frac{3}{4} - 2\frac{2}{3}$. **11.** $5\frac{3}{4} - 2\frac{5}{6}$. **12.** $8\frac{1}{6} - 3\frac{2}{3} \times 1\frac{1}{2}$.

13. If it takes $5\frac{5}{8}$ yards of cloth for one suit of clothes, how many yards will make 9 suits?

14. $9\frac{1}{4} - 7\frac{7}{8} \div 4\frac{1}{5}$. **15.** $178\frac{3}{4} \div 5$. **16.** $8\frac{7}{8} + 5\frac{2}{5} \times 4\frac{1}{6}$.

17. A merchant sold $26\frac{7}{8}$ yards of lace in equal parts to 5 ladies. How many yards did each buy?

18. $9\frac{3}{4} - 2\frac{2}{3} \times 2\frac{1}{2}$. **19.** $43\frac{5}{7} \times 40$. **20.** $8\frac{3}{4} \div 2\frac{5}{8} - 1\frac{5}{6}$.

21. A man received $38 for $\frac{5}{7}$ of his berries at 8 cents a quart. How many quarts did he have left?

22. $6\frac{2}{3} \div \overline{7\frac{1}{2} - 4\frac{2}{3}}$. **23.** $86 \times 50\frac{4}{5}$. **24.** $9\frac{1}{6} - \overline{3\frac{3}{4} - 2\frac{2}{3}}$.

25. A clerk spends $700 a year and has $\frac{4}{9}$ of his salary left. How much of his salary does he save?

26. $21\frac{5}{6} + 8\frac{3}{4} \div 2\frac{5}{8} - 14\frac{5}{6}$. **27.** $4\frac{1}{6} \times 1\frac{4}{5} - 8\frac{2}{7} \div 2\frac{2}{7}$.

28. A man sold 75 acres of land for $6375 and lost $\frac{3}{19}$ as much as it cost. Find the cost per acre.

29. A man sold $\frac{1}{2}$ of his wheat to one man, $\frac{1}{3}$ to another, and had 168 bushels left. How much did he sell?

30. A man sold 1 fifteenth of his wheat for $85.65. At the same price, how much is half the remainder worth?

DECIMALS

When anything is divided into tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, or millionths, the parts are called *decimal parts.*

A **Decimal Fraction,** or **Decimal,** is one or more of the equal decimal parts of anything.

$$111 = 100 + 10 + 1$$

The first 1 at the right of 111 denotes 1; the second, 10; the third, 100. Each 1 therefore denotes 1 tenth of the value of the 1 next at the left. If we place a period, called the *decimal point,* after the first 1, a 1 immediately at the right of the decimal point denotes 1 *tenth;* the next 1, 1 tenth of 1 tenth, or 1 *hundredth;* the next 1, 1 tenth of 1 hundredth, or 1 *thousandth;* etc. The figures at the right of the decimal point are called *decimal places.* We see therefore that the first decimal place expresses *tenths;* the second, *hundredths;* the third, *thousandths;* the fourth, *ten-thousandths;* and so on.

A **Mixed Decimal** is a whole number and a decimal written together.

Read *and* only at the decimal point.

Read: 4.6 17.005 208.065 600.06 8025.008

A **Complex Decimal** is a decimal with a common fraction written at its right.

Read: $.15\frac{2}{3}$ $.5\frac{1}{3}$ $.008\frac{5}{6}$ $.06\frac{1}{9}$ $.0009\frac{1}{7}$

To write decimals, it is only necessary to remember that one decimal place expresses tenths; two places, hundredths; three places, thousandths; four places, ten-thousandths; five places, hundred-thousandths; etc.

$\frac{7}{10} = \frac{70}{100}$ $.7 = .70$ $\frac{130}{1000} = \frac{13}{100}$ $.130 = .13$

PRINCIPLE. — *Annexing ciphers to a decimal does not change its value; and conversely, rejecting ciphers from the right of a decimal does not change its value.*

REDUCING DECIMALS TO COMMON FRACTIONS

$.24 = \frac{24}{100} = \frac{6}{25}$ $.85 = \frac{85}{100} = \frac{17}{20}$ $.38 = \frac{38}{100} = \frac{19}{50}$

RULE. — *Express the decimal in the form of a common fraction and reduce it to its lowest terms.*

1. Reduce to common fractions: .18, .425, .025, .0125.
2. Reduce to common fractions: .26, .345, .072, .0075.
3. Reduce to common fractions: .85, .175, .008, .0625.

REDUCING COMMON FRACTIONS TO DECIMALS

$$\frac{8}{25} = \frac{8 \times 4}{25 \times 4} = \frac{32}{100} = .32 \qquad \frac{37}{750} = \frac{37 \times 1\frac{1}{3}}{750 \times 1\frac{1}{3}} = \frac{49\frac{1}{3}}{1000} = .049\tfrac{1}{3}$$

RULE. — *Multiply both terms of the fraction by a number that will make the denominator a power of* 10 *and express the result in the decimal form.*

If the denominator of the fraction is expressed by two figures, reduce the fraction to hundredths; if it is expressed by three figures, reduce the fraction to thousandths; if it is expressed by four figures, reduce the fraction to ten-thousandths; etc.

1. Express in the decimal form: $\frac{7}{80}$, $\frac{19}{40}$, $\frac{27}{125}$, $\frac{9}{4000}$.
2. Express in the decimal form: $\frac{9}{40}$, $\frac{17}{75}$, $\frac{51}{400}$, $\frac{111}{1250}$.
3. Express in the decimal form: $\frac{7}{60}$, $\frac{13}{80}$, $\frac{37}{300}$, $\frac{79}{3000}$.

When it is difficult to determine what number will reduce the fraction to hundredths, thousandths, or ten-thousandths, the fraction may be reduced to a decimal by the following:

Annex ciphers to the numerator and divide by the denominator, and from the right of the quotient point off as many decimal places as there are ciphers annexed.

1. Express in the decimal form: $\frac{3}{16}$, $\frac{23}{47}$, $\frac{67}{137}$, $\frac{31}{2375}$.
2. Express in the decimal form: $\frac{9}{35}$, $\frac{49}{67}$, $\frac{45}{128}$, $\frac{9}{1370}$.

ADDITION AND SUBTRACTION OF DECIMALS

To add and subtract decimals, write units of the same order in a column and proceed as with whole numbers.

2.45	42.7	5.5	125.75	36.136
1.7	6.25	45.68	40.6	142.023
5.06	38.075	286.375	7.135	65.009
9.21	87.025	337.555	173.485	243.168

1. Add 8.35, 57.045, 8.6, .625, 65.8, 584.87, and 8.42.
2. Add 85.7, 648.85, 8.4, 68.3, 7.65, 42.125, and 88.6.
3. Add 235.007, 68.94, 7.6, 28.4, 7.36, 75.2, and 7.34.

8.64	45.6	502.4	180.05	246.008
3.8	9.125	48.625	24.075	25.125
4.84	36.475	453.775	155.975	220.883

1. 8.24 – 5.075. 2. 35.5 – 9.65. 3. 32.015 – 6.4.
4. 52.4 – 18.85. 5. 61.06 – 7.5. 6. 40.5 – 5.045.
7. 60.2 – 8.405. 8. 80.5 – .875. 9. 50.06 – .486.

MULTIPLICATION OF DECIMALS

5.38 × 12 means, find 12 times 5.38.

1. 5.38 × 12.	**2.** .446 × 24.	**3.** .075 × 48.
5.38	.446	.075
12	24	48
10.76 = 2 times 5.38	1.784 = 4 times .446	.600
53.8 = 10 times 5.38	8.92 = 20 times .446	3.00
64.56 = 12 times 5.38	10.704 = 24 times .446	3.600

Be careful to place the decimal point in each partial product and in the final result *immediately* after writing tenths.

1. 465.8 × 26. **2.** 7.643 × 34. **3.** 57.38 × 46.

4. 73.47 × 35. **5.** 581.7 × 56. **6.** 74.35 × 64.

7. 8.276 × 45. **8.** 86.75 × 48. **9.** 4.548 × 75.

844 × .8 means, find $\frac{8}{10}$ of 844.

1. 844 × .8.	**2.** 63.5 × .7.	**3.** 9.65 × .9.
84.4 = $\frac{1}{10}$ of 844	6.35 = $\frac{1}{10}$ of 63.5	.965
8	7	9
675.2 = $\frac{8}{10}$ of 844	44.45 = $\frac{7}{10}$ of 63.5	8.685

1. 687.6 × .8. **2.** 67.59 × .6. **3.** 9.457 × .7.

4. 456.5 × .4. **5.** 59.68 × .9. **6.** 8.775 × .8.

4625 × .06 means, find $\frac{6}{100}$ of 4625.

1. 4625 × .06.	**2.** 386.5 × .07.	**3.** 69.75 × .08.
46.25 = $\frac{1}{100}$ of 4625	3.865 = $\frac{1}{100}$ of 386.5	.6975
6	7	8
277.50 = $\frac{6}{100}$ of 4625	27.055 = $\frac{7}{100}$ of 386.5	5.5800

1. 674.8 × .07.
2. 5.376 × .24.
3. 845.7 × .008.
4. 93.65 × .36.
5. 87.5 × .108.
6. 63.75 × .048.
7. 8.125 × .84.
8. 76.45 × .56.
9. 9.248 × .875.

$1.6 = \frac{16}{10}$

$4.05 = \frac{405}{100}$

$3.006 = \frac{3006}{1000}$

1. 48.2 × 1.6.
2. 321.5 × 4.05.
3. 2124 × 3.006

$4.82 = \frac{1}{10}$ of 48.2
16
28.92
48.2
$77.12 = \frac{16}{10}$ of 48.2

$3.215 = \frac{1}{100}$ of 321.5
405
16.075
1286.0
$1302.075 = \frac{405}{100}$ of 321.5

2.124
3006
12.744
6372.
6384.744

1. 65.8 × 8.6.
2. 465.7 × 3.25.
3. 734.5 × 2.106.
4. 7.36 × 4.5.
5. 63.98 × 5.36.
6. 57.48 × 4.375.
7. .875 × 8.4.
8. 9.465 × 7.28.
9. 8.875 × 7.384.

DIVISION OF DECIMALS

25.5 ÷ 15 means, find $\frac{1}{15}$ of 25.5.

25.5(15
15 1.7
10 5
10 5

2778.4(46
276 60.4
18 4
18 4

3.9(15
3 0.26
90
90

.884(26
78 .034
104
104

Pupils should be very careful to place the decimal point in the quotient after using the unit figure of the dividend.

1. 329.35 ÷ 7.
2. 1450.8 ÷ 18.
3. .68496 ÷ 8.
4. 7.178 ÷ 36.
5. 289.92 ÷ 48.
6. 11.31 ÷ 26.
7. 991.2 ÷ 14.
8. 1.7204 ÷ 34.
9. 547.4 ÷ 68.

In dividing by a decimal, we first reduce dividend and divisor to the smallest decimal unit in the divisor, and then divide the number of decimal units in the dividend by the number of like units in the divisor.

68.64 ÷ 2.4.	15.3 ÷ 4.08.	357 ÷ 2.04.	313 ÷ 5.008.
686.4(24	1530(408	35700(204	313000(5008
48 28.6	1224 3.75	204 175	30048 62.5
206	3060	1530	12520
192	2856	1428	10016
14 4	2040	1020	25040
14 4	2040	1020	25040

1. 122.5 ÷ 2.8. 2. 23.4 ÷ 6.24. 3. 262.8 ÷ 7.008.
4. 11.04 ÷ 6.4. 5. 39.9 ÷ 4.75. 6. 99.13 ÷ 1.724.
7. 8.145 ÷ 7.5. 8. 91.8 ÷ 4.25. 9. 7.515 ÷ 1.875.

1. A man raised 1275 quarts of berries and sold .84 of them. How many quarts did he have left?

2. A man had $8500. He bought a farm and had .16 of his money left. Find the cost of the farm.

3. A man sold .65 of his wheat and had 294 bushels left. How many bushels did he raise?

4. A man bought .8 of a ton of coal for $4.40. At the same price, what would 5.45 tons cost?

5. A lady spent .38 of her money for a cloak that cost $95. How much money did she have left?

6. If a man receives $2.80 for .8 of a day's work, how much will he receive, if he works 12.4 days?

When one of two numbers is a certain fractional part of 100 or of 1000, of $1 or of $10, the product of the two numbers is more easily determined than by the usual process of multiplication.

$25 = \frac{1}{4}$ of 100. **$75 = \frac{3}{4}$ of 100.** **$12\frac{1}{2} = \frac{1}{8}$ of 100.**

$37\frac{1}{2} = \frac{3}{8}$ of 100. **$62\frac{1}{2} = \frac{5}{8}$ of 100.** **$87\frac{1}{2} = \frac{7}{8}$ of 100.**

$100 \times 28 = 2800$, hence $25 \times 28 = \frac{1}{4}$ of 2800, or 700.

1. 25×24.
2. $12\frac{1}{2} \times 32$.
3. $37\frac{1}{2} \times 28$.
4. 75×36.
5. $62\frac{1}{2} \times 28$.
6. $87\frac{1}{2} \times 48$.
7. 25×26.
8. $12\frac{1}{2} \times 36$.
9. $37\frac{1}{2} \times 54$.

$\$.25 = \$\frac{1}{4}$. **$\$.75 = \$\frac{3}{4}$.** **$\$.12\frac{1}{2} = \$\frac{1}{8}$.** **$\$.37\frac{1}{2} = \$\frac{3}{8}$.**

$\$1 \times 96 = \$96$, hence $\$.12\frac{1}{2} \times 96 = \frac{1}{8}$ of $96, or $12.

1. $\$.25 \times 24$.
2. $\$.12\frac{1}{2} \times 48$.
3. $\$.37\frac{1}{2} \times 24$.
4. $\$.75 \times 32$.
5. $\$.62\frac{1}{2} \times 32$.
6. $\$.87\frac{1}{2} \times 48$.
7. $\$.25 \times 27$.
8. $\$.12\frac{1}{2} \times 36$.
9. $\$.37\frac{1}{2} \times 56$.

$125 = \frac{1}{8}$ of 1000. **$250 = \frac{1}{4}$ of 1000.** **$375 = \frac{3}{8}$ of 1000.**

$56 \times 1000 = 56{,}000$, $56 \times 625 = \frac{5}{8}$ of 56,000, or 35,000.

1. 125×24.
2. 250×28.
3. 375×32.
4. 625×48.
5. 750×32.
6. 375×56.
7. 125×64.
8. 375×56.
9. 625×72.

$\$1.25 = \frac{1}{8}$ of $10. **$\$2.50 = \frac{1}{4}$ of $10.** **$\$3.75 = \frac{3}{8}$ of $10.**

$\$10 \times 56 = \$560$, $\$3.75 \times 56 = \frac{3}{8}$ of $560, or $210.

1. $\$1.25 \times 24$.
2. $\$2.50 \times 48$.
3. $\$3.75 \times 32$.
4. $\$6.25 \times 48$.
5. $\$7.50 \times 32$.
6. $\$8.75 \times 56$.
7. $\$1.25 \times 36$.
8. $\$2.50 \times 26$.
9. $\$3.75 \times 64$.

COST BY THE 100, 1000, OR TON

If 100 lb. of beef cost $8.60, 1 lb. costs 1 hundredth of $8.60, or $.086, and 875 lb. cost

$$\$.086 \times 875 = \$75.25$$

If 1000 ft. of lumber cost $24, 1 ft. costs 1 thousandth of $24, or $.024, and 8475 ft. cost

$$\$.024 \times 8475 = \$203.40$$

A man bought lumber at $28 per M, paying $234.50 for it. How many feet did he buy? 1 ft. cost $.028.

$$\$234.50 \div \$.028 = 8375$$

At $15 a ton, 1000 lb. of hay cost $7.50, and 1 lb. costs 1 thousandth of $7.50, or $.0075. 9480 lb. cost

$$\$.0075 \times 9480 = \$71.10$$

A man bought hay at $16 a ton, paying $75 for it. How many pounds did he buy? 1 lb. cost $.008.

$$\$75 \div \$.008 = 9375$$

1. Find the cost of 1365 pounds of beef at $6 per C. At $9 per C. At $6.75 per C.

2. Find the value of 6745 feet of lumber at $12 per M. At $16 per M. At $14.50 per M.

3. Find the value of 5760 feet of lumber at $18 per M. At $15.75 per M. At $16.75 per M.

4. Find the cost of 8275 pounds of coal at $6 a ton. At $7 a ton. At $8 a ton. At $5.80 a ton.

5. Find the value of 9340 pounds of hay at $12.75 per ton. At $16.25 per ton. At $18.50 per ton.

6. At $8.80 per thousand, how many brick can be bought for $83.38?

7. At $6.45 a hundred, how many pounds of beef can be bought for $95.46?

8. A man bought 4 tons 8 cwt. 85 lb. of coal at $5.75 a ton. What did it cost?

9. A builder paid $416.40 for lumber at $24 a thousand feet. How many feet did he buy?

10. Find the value of 12,780 feet of lumber at $17 per M. At $25 per M. At $28 per M.

11. A man sold 6 tons 17 cwt. 60 lb. of straw at $9.75 a ton. What did he receive for it?

12. Find the value of 13,740 pounds of hay at $15 a ton. At $18.50 a ton. At $19 a ton.

13. A man sold hay at $13.50 a ton, receiving $116.10 for it. How many pounds did he sell?

14. Find the value of 14,360 pounds of hay at $11 a ton. At $12.75 a ton. At $14.50 a ton.

15. Find the cost of 7340 feet of lumber at $18.75 per M. At $24.25 per M. At $26.75 per M.

16. Find the value of 6 loads of hay, the average weight being 2160 pounds, at $15.80 a ton.

17. A farmer sold 8 tons 13 cwt. 80 lb. of hay at $13.75 a ton. What did he receive for it?

18. A man received $79.20 for 8 loads of hay at $9 a ton. Find the average weight of the loads.

19. A man bought lumber at $28 a thousand feet, paying $251.02 for it. How many feet did he buy?

20. A dealer sold 16 bales of hay averaging 185 pounds at 85 cents per C. How much did he receive for it?

BOARD MEASURE

Lumber is usually bought and sold by the thousand feet; that is, a thousand board feet.

A **Board Foot** of lumber is a piece 1 foot square and 1 inch thick, or its equivalent.

NOTE. — In measuring boards less than 1 inch thick, a board foot is a piece 1 foot square without regard to thickness.

RULE. — *Multiply the length in feet by the width in feet and that product by the thickness in inches.*

NOTE. — In computing board measure, take no account of the thickness when it is less than 1 inch.

How many board feet are there in 75 joists 24 feet long, 10 inches wide, and 3 inches thick?

$$24 \times \frac{5}{6} \times 3 \times 75 = 4500.$$

When lumber is 8″ by 1½″, 6″ by 2″, or 4″ by 3″, the number of board feet in each piece is the same as the number of feet in length; when lumber is 12″ by 2″, 8″ by 3″, or 6″ by 4″, the number of board feet in each piece is twice as many as the number of feet in length; when lumber is 12″ by 3″, 9″ by 4″, or 6″ by 6″, the number of board feet in each piece is three times as many as the number of feet in length; etc.

1. How many board feet are there in 85 joists 30 ft. by 10 in. by 3 in.?

2. How many board feet are there in 75 pieces of lumber 24 ft. by 5 in. by 4 in.?

3. At $16 per M, what is the value of 185 planks 18 ft. long, 8 in. wide, and 2 in. thick?

4. At $28 per M, what is the value of 55 pieces of lumber 30 feet long and 8 inches square?

5. How many board feet are there in 84 pieces of lumber 24 feet long and 15 inches square at the ends?

CARPETING

Carpeting is sold by linear measure. It is cut into strips, and these are sewed together. The number of yards required for a room is the combined length of all the strips. In matching the pattern, it is nearly always necessary to cut all the strips, *except the first*, longer than the dimension of the room. Besides this waste in matching the pattern, it is often necessary to buy part of a strip more than is needed. For example, if five strips and part of another are needed, six strips must be bought.

If the strips are to run *lengthwise*, we determine the number of strips to be bought by dividing the *width* of the room by the width of the carpet. The length of the first strip cut is the length of the room, and the length of each of the other strips is the length of the room plus the waste in matching.

How many yards of carpet 27 in. wide will be required for a room 15 ft. 6 in. long and 13 ft. wide, if the strips run lengthwise and there is a waste of 9 in. on each strip except the first in matching?

27 in. = 2.25 ft. 13 ÷ 2.25 = 5 + 6 strips.
1 strip . . . . . 15′ 6″
5 strips, each 16′ 3″ . . . 81′ 3″

The sum of the 6 strips is 96¾ feet, or 32¼ yards.

If the strips are to run *across* the room, we determine the number of strips to be bought by dividing the *length* of the room by the width of the carpet.

15.5 ÷ 2.25 = 6 + 7 strips.
1 strip . . . . . 13′
6 strips, each 13′ 9″ . . . 82′ 6″

1. How many yards of carpet 1 yd. wide will be required for a room 17 ft. 4 in. by 14 ft. 10 in., if the strips run lengthwise and there is a waste of 6 in. on each strip except the first in matching? How many yards will be required, if the strips run across the room?

2. How many yards of carpet 27 in. wide will be required for a room 16 ft. 9 in. long and 15 ft. wide, if the strips run lengthwise and there is a waste of 10 in. on each strip except the first in matching? How many yards will be required, if the strips run across the room?

3. How many yards of carpet 27 in. wide will be required for a room 22 ft. 9 in. by 17 ft. 8 in., if the strips run lengthwise and there is a waste of 8 in. on each strip except the first in matching? How many yards will be required, if the strips run across the room?

4. At $45 per M, what is the value of 26 pieces of lumber 24 feet long and 18 inches square?

5. How many feet of lumber will board up to the eaves the sides and ends of a barn 78 feet long and 38 feet wide, if the barn is 22 feet high to the eaves?

6. At $35 per C for the posts and $26 per M for the boards, what will it cost to build a fence four boards high around a rectangular field 64 rd. by 48 rd., setting the posts 8 feet apart and using boards 10 in. wide?

7. A man built a close board fence 48 rods long and 4 feet high. The boards were in an upright position and were nailed to two scantlings 4 by 2 inches, and the posts were set 8 feet apart. If he paid $32 per C for the posts, $18 per M for the scantlings, and $24 per M for the boards, what did all the material for the fence cost?

CAPACITY OF CISTERNS

Since a gallon contains 231 cubic inches, the capacity of a cistern may be determined by finding the cubic contents in cubic inches and dividing the result by 231. A more convenient method is to multiply the cubic contents in cubic feet by 7.5 and subtract from the result $\frac{1}{4}$ of $\frac{1}{100}$ of itself. This method will give the capacity of a large cistern correct to a very small fraction of a gallon.

How many gallons will a cistern hold that is 16 feet long, 7 feet wide, and 6 feet deep?

$$16 \times 7 \times 6 \times 7.5 = 5040$$

$$\tfrac{1}{100} \text{ of } 5040 = 50.4$$

$$5040 - \tfrac{1}{4} \text{ of } 50.4 = 5027.4$$

The capacity as determined by the longer process is 5026.9 gallons, which is .5 of a gallon less than the first result.

To find the capacity of a cistern in barrels of $31\frac{1}{2}$ gallons, *divide the number of cubic feet by* 4.21.

1. How many barrels of water will a cubical cistern hold that is 12 feet deep?

2. Find the capacity in gallons of a cistern 8 feet square and 5 feet 6 inches deep.

3. Find the capacity in gallons of a cistern 8 feet long, 6 feet wide, and 4 feet 6 inches deep.

4. How many gallons of water will a cistern hold that is 12 feet long, 7 feet wide, and 6 feet deep?

5. Find the capacity in gallons of a cistern that is 13 feet square and 7 feet 6 inches deep.

6. How many gallons will a cistern hold that is 14 ft. 6 in. long, 7 ft. 4 in. wide, and 5 ft. 3 in. deep?

CAPACITY OF BINS

Since a bushel contains 2150.42 cubic inches, the capacity of a bin may be determined by finding the cubic contents in cubic inches, and dividing the result by 2150.42. A more convenient method is to multiply the cubic contents in cubic feet by .8 and add to the result $\frac{1}{2}$ of $\frac{1}{100}$ of itself. This method will give the capacity of a large bin correct to a very small fraction of a bushel.

How many bushels of wheat will a bin hold that is 15 feet long, 8 feet wide, and 7 feet deep?

$$15 \times 8 \times 7 \times .8 = 672$$

$$\tfrac{1}{100} \text{ of } 672 = 6.72$$

$$672 + \tfrac{1}{2} \text{ of } 6.72 = 675.36$$

The capacity as determined by the longer process is 674.99 bushels, which is .37 of a bushel less than the first result.

1. How many bushels of wheat will a cubical bin hold that is 12 feet deep?

2. How many bushels of oats will a bin hold that is 12 feet square and 8 feet deep?

3. Find the capacity in bushels of a bin 11 ft. 3 in. by 7 ft. 4 in. by 5 ft. 6 in.

4. A bin 10 feet square and 6 feet deep is full of wheat. How many bushels are there?

5. How many bushels of rye can a man put into a cubical bin that is 11 feet deep?

6. Find the capacity in bushels of a bin 12 feet 6 inches long, 8 feet wide, and 7 feet 6 inches deep.

7. Find the capacity in bushels of a bin 14 feet 6 inches long, 8 feet wide, and 6 feet 6 inches deep.

PERCENTAGE

The phrase, *per cent*, and the symbol, %, mean hundredths. 5% is $\frac{5}{100}$. 15% is $\frac{15}{100}$. 4.8% is $\frac{4.8}{100}$, etc.

FIRST TYPE OF QUESTION IN PERCENTAGE

Find 18% of 425.

$$\begin{array}{l} 4.25 = 1\% \text{ of } 425 \\ \underline{18} \\ 34.00 \\ \underline{42.5} \\ 76.50 = 18\% \text{ of } 425 \end{array}$$

$7\frac{1}{2}$% of 468.

$$\begin{array}{l} 4.68 = 1\% \text{ of } 468 \\ \underline{7\tfrac{1}{2}} \\ 2.34 \\ \underline{32.76} \\ 35.10 = 7\tfrac{1}{2}\% \text{ of } 468 \end{array}$$

48% of 6500.

$$\begin{array}{r} 65 \\ \underline{48} \\ 520 \\ \underline{260} \\ 3120 \end{array}$$

Since per cent means hundredths, 100% of a number is the whole of it; 200% of a number is twice the number, and so on.

10% = $\frac{1}{10}$.
20% = $\frac{1}{5}$.
40% = $\frac{2}{5}$.
60% = $\frac{3}{5}$.

80% = $\frac{4}{5}$.
25% = $\frac{1}{4}$.
50% = $\frac{1}{2}$.
75% = $\frac{3}{4}$.

$16\frac{2}{3}$% = $\frac{1}{6}$.
$83\frac{1}{3}$% = $\frac{5}{6}$.
$33\frac{1}{3}$% = $\frac{1}{3}$.
$66\frac{2}{3}$% = $\frac{2}{3}$.

$12\frac{1}{2}$% = $\frac{1}{8}$.
$37\frac{1}{2}$% = $\frac{3}{8}$.
$62\frac{1}{2}$% = $\frac{5}{8}$.
$87\frac{1}{2}$% = $\frac{7}{8}$.

25% of 64 is $\frac{1}{4}$ of 64, or 16. $16\frac{2}{3}$% of 96 is $\frac{1}{6}$ of 96, or 16. $12\frac{1}{2}$% of 56 is $\frac{1}{8}$ of 56, or 7. $33\frac{1}{3}$% of 75 is 25.

$\frac{3}{4}$% of 6400 = $\frac{3}{4}$ of 64, or 48.

1. Find 55% of 980 children. 68% of 1350 bushels. 135% of $1640. 8.5% of $2400. $4\frac{3}{4}$% of $960.

2. A farmer raised 940 bushels of potatoes and sold 85% of them. How many bushels did he have left?

3. A man had $250,000 to invest. He invested 16% of it in railroad bonds, 45% of it in government bonds, and the remainder in real estate. Find each investment.

SECOND TYPE OF QUESTION IN PERCENTAGE

245 is 7% of what number? $45.36 is 6% of what?

7)245.00
35.00
3500

Since 245 is $\frac{7}{100}$ of some number, $\frac{1}{100}$ of the number is 35, and the whole number is 3500.

6)$45.36
7.56
$756

1. Find the number of which 310 is 8%. Of which 143.2 is 16%. Of which 4536 is 108%.

2. A man bought a horse for $162, which was 24% of his money. How much did he have left?

When any number is increased by 15% of itself, the result is 115% of that number. When any number is diminished by 35% of itself, the result is 65% of that number.

3. What number increased by 14% of itself equals 969? By 8%, equals 1944? By 35%, equals 9558?

4. What number diminished by 8% of itself equals 414? By 5%, equals 3857? By 28%, equals $466.56?

5. A boy spent $1.76, which was $7\frac{1}{8}$% of his money. How much money did he have left?

6. A lady spent 28% of her money for a cloak that cost $49. How much money did she have left?

7. After a man's salary had been increased 15%, he received $4600. How much was it increased?

8. A farmer lost 32% of his wheat by fire and had 1649 bushels left. How many bushels did he lose?

9. The number of boys in a school is 255, which is 34% of all the pupils. How many girls are there?

10. What number diminished by $17\frac{1}{2}$% of itself equals 732.6? By 93%, equals 339.5? By .7%, equals 47,664?

11. A man sold 891 lb. of wool, which was equal to 44% of what he had left. How many pounds had he at first?

THIRD TYPE OF QUESTION IN PERCENTAGE

If 1% of a number is 6, 24 is 4% of that number, because 24 is 4 times 6. If 1% of a number is 8, 36 is 4½% of that number, because 36 is 4½ times 8. If 7.5 is 1% of a number, 60 is 8% of that number, because 60 is 8 times 7.5.

76 is what per cent of 950?

1% of 950 is 9.5. $76 \div 9.5 = 760 \div 95 = 8$.

1. What per cent of 840 is 126? Of 1860 is 651? Of 3850 is 308? Of $560 is $35?

2. What per cent of 875 is 210? Of 450 is 477? Of 18,000 is 45? Of $1500 is $110?

3. A man had 240 sheep and sold 84 of them. What per cent of his flock did he have left?

4. In a school there are 153 boys and 272 girls. What per cent of the whole number are the girls?

5. A man raised 252 bu. of wheat from 27 bu. of seed. What per cent of the seed was the increase?

6. A merchant bought goods for $1400 and sold them for $1631. What per cent of the cost was the profit?

7. A farmer raised 4350 bushels of oats and sold 1392 bushels. What per cent of his crop did he have left?

8. A farmer sold 759 bushels of wheat and had 891 bushels left. What per cent of his crop had he left?

9. From a cask containing 35 gal. of wine, all but 4 gal. 3 qt. was sold. What per cent of the wine remained?

It is very important that pupils should see now that there are only three types of questions in percentage, which are represented by the following: What is 12% of 450? 76 is 8% of what number? 26.25 is what per cent of 700? One of these forms is involved in the solution of every problem in percentage, and pupils should be trained to determine which form is involved in the given problem before they attempt its solution.

1. A man discharged 19½% of a debt by paying $30.42. How much did he still owe?

2. A farmer sold 104 sheep and had 221 left. What per cent of his flock did he sell?

3. I drew out 44% of my money in the bank and had $2702 left in the bank. Find the sum drawn out.

4. There are 490 girls in a school of 875 pupils. What per cent of the whole number are the boys?

5. A boy lost 66 marbles, which is equal to 24% of the number he had left. How many had he at first?

6. The population of a city has increased 135 % in 15 years, and now it is 66,740. Find the increase.

7. A farmer lost 16% of his sheep, sold 40% of the remainder, and had 441 left. How many did he sell?

8. A man bought a horse for $575, which is 130% more than he paid for a carriage. Find the cost of both.

9. A received 48% of his father's estate and B 36% of it, A receiving $19,200 more than B. Find A's share.

10. A man paid $1139 on a debt and still owes $402 more than he paid. What % of the debt does he owe?

11. A man engaging in business lost 14% of his money the first year and 8% of the remainder the second year. He then had $11,472.40. How much did he lose?

12. A man bought property, which increased in value 25% the first year and 32% of its increased value the second year, when it was worth $11,220. Find the cost.

13. A man willed 40% of his property to his wife, 50% of the remainder to his son, 60% of what still remained to his daughter, and the remainder, amounting to $90,000, to a college. How much did the daughter receive?

PROFIT AND LOSS

Profit or **Loss** is the difference between the cost and selling price. If the selling price is greater than the cost, the difference is the profit; if the cost is greater than the selling price, the difference is the loss.

In questions in profit and loss, the gain or loss is usually computed at a certain per cent on the cost.

1. A man bought land for $1500 and sold it at a profit of 16%. How much did he gain?

Question: What is 16% of $1500?

2. By selling goods at a profit of 15%, a merchant gained $62. How much did the goods cost?

Question: $62 is 15% of what sum?

3. A dealer sold fruit at a profit of 14%, receiving $513 for it. What did the fruit cost?

Question: $513 is 114% of what sum?

4. A speculator sold land for $7040, which was 12% less than he paid for it. Find the cost.

Question: $7040 is 88% of what sum?

5. A man bought apples for $175 and sold them for $217. Find the gain per cent.

Question: $42 is what per cent of $175?

The questions indicated above are the first questions to be answered in such problems, although the problem may contain a different question. For example, we might be required to find the selling price in the first and second, the gain in the third, the loss in the fourth.

1. A stationer bought books for $186 and sold them at a profit of 35%. Find the selling price.

2. There was a profit of 18% on a farm sold for $8850. Find the cost and gain.

3. A merchant sold his household furniture for $1196 and lost $184. Find the loss per cent.

4. A merchant lost $6\frac{2}{3}$% by selling rubbers at 70 cents a pair. Find the loss on each pair.

5. A merchant sold a bill of goods for $276 and made a profit of $36. Find the gain per cent.

6. A man sold a carriage at a loss of $27.60, which was 23% of the cost. Find the selling price.

7. If there is a loss of 8% on goods sold at 46 cents a yard, what is the loss per yard?

8. By selling a house for $975 more than it cost, a man gained 15%. Find the selling price.

9. A dealer gained $18\frac{3}{4}$% on shoes for which he received $4.75 a pair. Find the profit on each pair.

10. A merchant lost 24% on some damaged goods for which he paid $875. Find the selling price.

11. A man sold his house for 128% of the cost and gained $728. How much did he receive for it?

12. A man sold goods for $113\frac{1}{2}$% of their cost and made a profit of $49.14. What did he receive for them?

13. A merchant lost $21.25 by selling some silk for $91\frac{1}{2}$% of the cost. Find the sum received for it.

14. A grocer sold tea at 80 cents a pound, which was 15 cents more than it cost. Find the gain per cent.

15. A man sold a watch and chain for $147, which was $33 less than he paid for them. Find the loss per cent.

1. A man sold a carriage for $252, which was 28% less than he paid for it. Find the loss.

2. A man sold a horse for $560, which was 40% more than he paid for the horse. Find the gain.

3. A piece of land was sold for $5400, which was 116% more than it cost. Find the profit.

4. A grocer gained $9.45 by selling 63 gal. of molasses at 60 cents a gallon. Find the gain per cent.

5. A man sold real estate for 92¾% of the cost and lost $319. How much did he receive for it?

6. A merchant sold goods for $38.50 less than he paid for them, losing 14%. What did he receive for them?

7. A man offered his house for sale at 15% below cost, but afterward sold it for $7497, which is 5% more than his first offer. How much did he lose?

8. A speculator refused $20,700 for wool, because he would lose 8% by selling it for that sum. He afterward sold it for $23,475. Find the gain per cent.

9. A man sold a carriage at a profit of 33⅓%, and with the money bought another, which he sold for $180 and lost 25%. Find the profit on the first one.

10. A builder sold two houses for $2400 each, gaining 20% on one of them and losing the same per cent on the other. Did he gain or lose on both and how much?

11. A man offered his farm for sale at 15% below cost, but afterward sold it for $6460, which was 5% less than his first offer. How much did he lose?

12. A man sold a horse for $150 and lost 40%. He then sold another at a profit of 16⅔% and gained as much as he lost on the first. Find the cost of each horse.

COMMISSION

A **Commission Merchant**, or **Agent**, is a person who buys and sells goods or does business for another.

The **Commission** is the sum received by an agent for doing business for another.

The **Proceeds** of a sale is the sum left after the commission and other charges have been taken out.

The agent's commission is computed at a certain per cent on the amount of the sale or the sum invested.

1. A commission merchant sold 600 bu. of potatoes at $.75 a bushel on 5% commission. Find the proceeds.

$\$450 \times .05 = \$22.50. \qquad \$450 - \$22.50 = \$427.50.$

2. An agent received $225 for selling a house on 5% commission. Find the proceeds of the sale.

$\$225 \div .05 = \$4500. \qquad \$4500 - \$225 = \$4375.$

3. An agent received $8652 to invest in wool after taking out his commission of 5%. Find the sum invested.

Question: $8652 is 105% of what sum?

4. The proceeds of a sale were $418 after taking out the commission of 5%. Find the amount of the sale.

Question: $418 is 95% of what sum?

5. The proceeds of a sale were $439.30 and the commission $20.70. Find the rate of commission.

Question: $20.70 is what per cent of $460?

In the third and fourth problems, we might be required to find the commission. In the third, the commission is $8652 less the sum invested; in the fourth, the commission is the amount of the sale less $418.

1. A manufacturer paid his agent $129.51 for selling goods on 4½% commission. Find the proceeds.

2. The proceeds of a sale on 8% commission were $693.45. Find the agent's commission.

3. An auctioneer sold furniture for $1250 on 5½% commission. Find the proceeds of the sale.

4. The proceeds of a sale were $401.10 and the commission $18.90. Find the rate of commission.

5. An agent received $48.60 for selling butter on 4½% commission. Find the proceeds of the sale.

6. If the proceeds of a sale are $798 and the commission $42, what is the rate of commission?

7. An agent received $7119 to invest in wool after taking out his commission of 5%. Find the commission.

8. The proceeds of a sale of apples were $822.50 and the commission $52.50. Find the rate.

9. An agent received $161.50 for selling real estate on 5% commission. Find the proceeds of the sale.

10. A man received $250.99 for butter, after a commission of $13.21 had been taken out. Find the rate.

11. A miller paid his agent $143.80 for selling flour on 5% commission. Find the proceeds of the sale.

12. The proceeds of a sale were $4777.50 and the rate of commission 2%. Find the agent's commission.

13. An agent received $3159 to invest in flour after taking out his commission of 4%. Find the commission.

14. The sum left from a sale, after deducting the commission of 5%, was $326.80. What was the commission?

15. A commission merchant received $1680 for selling wool on 4% commission. Find the proceeds of the sale.

1. A commission merchant received $39.30 for selling produce on 5% commission. Find the proceeds.

2. An agent sold a section of land at $22 an acre and received a commission of 4½%. Find the proceeds.

3. A merchant received $854.40 for goods which an agent sold on 4% commission. Find the commission.

4. A commission merchant sold 450 barrels of flour at $4.60 a barrel and remitted to the consignor $1966.50. Find the rate of commission.

5. An agent received $34,000 to invest in wool after taking out his commission of 2½% and $277.50 to pay freight. Find the agent's commission.

6. A farmer received $1197 for apples, which a commission merchant sold for him at $2.25 a barrel on 5% commission. How many barrels did he sell?

7. An agent sold produce, and after taking out his commission of 5% and $16.50 each for freight and storage, he remitted $1240. Find the commission.

8. A commission merchant received $400 from a sale of apples. He invested the proceeds in other fruit, reserving 2½% for buying, 5% for selling, and $21.25 for other charges. Find the whole commission.

9. An agent sold a house and invested the proceeds in land after taking out his commission of 5% for selling the house and 4% for buying the land. The agent paid $4750 for the land. Find the whole commission.

10. After taking out his commission of 4% and $40.40 for other charges, a commission merchant remitted to a farmer $910 for potatoes sold at $.60 a bushel. Find the commission and the number of bushels sold.

INSURANCE

Companies are organized to engage in the business of insurance. If a man pays a certain sum of money to an insurance company, the company enters into an agreement to pay him a specified sum, if his property is destroyed by fire within a certain limited time. The contract given to the man by the company is called the ***insurance policy***, and the sum paid for the insurance is called the ***premium***.

1. A house worth $6000 is insured for ¾ of its value at 75 cents a $100. What is the premium?

Question: What is ¾% of $4500?

2. A man paid $60 for insuring his store at 1¼%. For what sum was the store insured?

Question: $60 is 1¼% of what sum?

The **Sum Covered by Insurance** is the sum insured less the premium.

3. For what sum must a stock of goods be insured at 1¼% to cover a loss of $8295?

Question: $8295 is 98¾% of what sum?

4. A church valued at $60,000 is insured for ⅘ of its value at 1¼%. Find the premium.

5. A man insured his store at 80 cents a $100, paying a premium of $56. Find the sum insured.

6. For what sum must a man get his library insured at 75 cents a $100 to cover a loss of $595.50?

7. It cost $625 to insure a stock of goods for their full value at 2½%. Find the sum covered by insurance.

1. What will it cost to insure goods worth $8750 for $\frac{4}{5}$ of their value at 75 cents a $100?

2. For what sum must goods worth $16,590 be insured at $1\frac{1}{4}\%$ to cover 5 sixths of their value?

3. It cost $360 to insure buildings for $\frac{4}{5}$ of their value at $.75 a $100. Find their value.

4. A house worth $6800 was insured for $\frac{3}{4}$ of its value, and the premium was $38.25. Find the rate.

5. What will it cost to insure property worth $16,500 for $\frac{2}{3}$ of its value at $.80 a $100?

6. A man insured his house for $\frac{3}{4}$ of its value at $1\frac{1}{4}\%$. The house was destroyed by fire, and his loss was $1245. Find the value of the house.

7. A merchant insured his store and goods for $\frac{5}{6}$ of their value at $1\frac{3}{4}\%$, paying a premium of $271.25. If they are destroyed by fire, what will he lose?

8. A merchant insured his store for $8000, or 4 fifths of its value, and his goods, valued at $18,000, for 8 ninths of their value, both at $1\frac{1}{4}\%$. The store and goods were totally destroyed by fire. What was his loss?

TAXES

A **Tax** is a sum of money raised for public purposes.

An **Assessment Roll** is a list of the names of taxable inhabitants, and the value of each person's property.

Assessors are officers appointed to estimate the value of taxable property.

A **Tax Roll** is a list of the names of the taxable inhabitants, with the tax of each.

Each person's tax is reckoned at a certain per cent on the value of his property as indicated on the assessment roll. To find the rate, divide the sum to be raised by the total valuation, carrying the division to six decimal places. Multiply the rate thus obtained by 2, 3, 4, 5, 6, 7, 8, and 9, and the several products will be the tax on the respective number of dollars. From these a table similar to the following is constructed from which each person's tax is easily computed.

RATE $.007154

$1	$.01	$10	$.07	$100	$0.72	$1000	$7.15
$2	.01	$20	.14	$200	$1.43	$2000	$14.31
$3	.02	$30	.21	$300	$2.15	$3000	$21.46
$4	.03	$40	.29	$400	$2.86	$4000	$28.62
$5	.04	$50	.36	$500	$3.58	$5000	$35.77
$6	.04	$60	.43	$600	$4.29	$6000	$42.92
$7	.05	$70	.50	$700	$5.01	$7000	$50.08
$8	.06	$80	.57	$800	$5.72	$8000	$57.23
$9	.06	$90	.64	$900	$6.44	$9000	$64.39

At this rate, find the tax on these sums: $3500, $2700, $4375, $48,625, $25,580, $76,460.

1. Prepare the table to be used in making the tax roll when the sum to be raised is $8750 and the total valuation of property on the assessment roll is $2,146,500.

2. Prepare the table to be used in making the tax roll when the sum to be raised is $47,280 and the total valuation of property on the assessment roll is $15,475,850.

INTEREST

When a man borrows money, he gives to the person lending it a written promise to pay the sum borrowed either on demand or at the end of a specified time. He also promises to pay a certain sum for the use of the money. This promise is generally similar to one of the following:

$475.85 NEWBURGH, N. Y., May 15, 1902.

On demand, I promise to pay George W. Davis, or order, Four Hundred Seventy-five and $\frac{85}{100}$ Dollars, with interest at 6%. Value received.

CHARLES E. PHILLIPS.

$475.85 NEWBURGH, N. Y., May 15, 1902.

Four months after date, I promise to pay George W. Davis, or order, Four Hundred Seventy-five and $\frac{85}{100}$ Dollars, with interest at 5%. Value received.

CHARLES E. PHILLIPS.

Such written promises are **Promissory Notes**. The sum borrowed is the **Principal**, or **Face** of the note; the sum paid for the use of the principal is the **Interest**; and the sum of the principal and interest is the **Amount**.

The **Maker** of a note is the person who signs it, and the **Payee** is the person to whom it is payable.

The interest that a man must pay on borrowed money is limited by law in most of the states. He may pay less than the legal rate, if a smaller rate is named in the note, but he cannot be compelled to pay more. The **Rate of Interest** is the interest on $1 for one year.

$375.50 NEWBURGH, N. Y., May 24, 1902.

One year from date, I promise to pay John W. Hamilton, or order, Three Hundred Seventy-five and $\frac{50}{100}$ *Dollars, with interest. Value received.*

JAMES A. BAKER.

What is the face of this note? Who is the maker of the note? Who is the payee?

SIX PER CENT METHOD

At 6%, the interest on $1 for 1 year is $.06; for 1 month it is $.005; for 6 days it is $.001.

Find the interest and amount of $650 from October 20, 1898, to June 8, 1902, at 6% per annum.

The time is 3 yr. 7 mo. 18 da.

The interest on $1 for 3 years is $.18; for 7 months it is $.035; for 18 days it is $.003. For the whole time it is $.218.

$.218 × 650 = $141.70. $650 + $141.70 = $791.70.

Since $.218 × 650 is the same as .218 of $650, we might find .218 of $650. This will be more convenient in many examples.

When the rate is other than 6%, we increase or diminish the interest at 6% by a certain fraction of itself.

3)$141.70 6%	4)$141.70 6%	6)$141.70 6%
47.23 2%	35.42 1½%	23.64 1%
$94.47 4%	$106.28 4½%	$118.06 5%

By *adding* the interest at 1%, 1½%, and 2% to the interest at 6%, we have the interest at 7%, 7½%, and 8%, respectively.

When the time is exact years, or years and an aliquot part of a year, multiply the yearly interest on a dollar by the principal and that product by the number of years.

1. Find the interest on $825.46 for 1 year 8 months 24 days at 6%.

2. Find the amount of $1264 for 4 yr. 6 mo. at 5%.

3. Find the interest on $1465 from June 6, 1890, to July 6, 1895, at 5%.

4. What is the amount of $265 for 25 days at 4%?

5. Find the interest on $484 from October 12, 1890, to May 12, 1892, at 7½%.

6. What is the amount of $875 for 2 yr. 3 mo. at 8%?

7. Find the interest on $308.75 for 2 years 2 months 14 days at 6%.

8. Find the interest on $575 from January 4, 1889, to March 16, 1892, at 6%.

9. Find the amount of $473.75 for 1 mo. 7 da. at 5½%.

10. Find the interest on $796 from May 25, 1890, to November 13, 1894, at 6%.

11. What is the amount of $1400 for 6 yr. 4 mo. at 3%?

12. Find the interest on $900 for 3 years 4 months 8 days at 4½%.

13. Find the amount of $1175 for 4 years 3 months 5 days at 6%.

14. Find the interest on $687.48 from January 14, 1884, to December 14, 1886, at 3%.

15. Find the amount of $580.48 from November 12, 1881, to January 18, 1887, at 6%.

16. Find the amount of $738 from April 15, 1889, to September 15, 1892, at 6½%.

17. Find the interest on $387.84 from September 18, 1890, to April 3, 1892, at 8%.

1. Find the interest on $4800 for 19 days at 6%.

2. Find the amount of $3867 for 1 year 6 months 2 days at 9%.

3. Find the amount of $2864 for 2 yr. 4 mo. at 7%.

4. Find the interest on $1748 from July 14, 1874, to April 14, 1879, at 6%.

5. Find the amount of $836 for 27 days at 7½%.

6. Find the interest on $428.40 from August 26, 1888, to December 5, 1890, at 3%.

7. Find the amount of $2450 for 5 yr. 6 mo. at 8%.

8. Find the interest on $264.80 from March 16, 1867, to November 16, 1872, at 4½%.

9. What is the amount of $1865 for 7 mo. 27 da. at 4%?

EXACT INTEREST

The usual method of computing interest for part of a year is not exact, for it is computed on the basis of 30 days in a month, or 360 days in a year.

A common year has 365 days and a leap year 366. The interest for any number of days is therefore that number of 365ths or 366ths of the interest for one year.

Find the exact interest on $2400 from March 15 to November 28 of a common year at 6%.

The time is 258 days. The interest for one year is $144. For 258 days it is $\frac{258}{365}$ of $144, which is $101.79.

If the time is more than a year, add the interest for the whole years, computed by the ordinary method, to the exact interest for the days, computed as above.

1. Find the exact interest on $742 for 145 days of a common year at 6%.

2. Find the exact interest on $4200 from January 12, 1891, to September 18, 1891, at 5%.

3. Find the exact interest on $1800 from November 8, 1889, to April 12, 1893, at 5%.

4. Find the exact interest on $28,500 from August 12, 1879, to June 15, 1880, at 7%.

5. Find the interest on $475.82 for 4 yr. 3 mo. at 4%.

6. Find the amount of $548 for 2 years 8 months 25 days at 6 per cent.

7. Find the interest on $764.48 for 3 mo. 22 da. at 5%.

8. Find the exact interest on $2800 from July 12, 1886, to June 2, 1890, at 5%.

9. Find the amount of $965.80 for 6 yr. 4 mo. at 7%.

10. Find the amount of $1470 for 4 years 4 months 12 days at 5½%.

11. Find the amount of $290.75 for 3 yr. 6 mo. at 5½%.

12. Find the amount of $798 from September 29, 1890, to April 17, 1895, at 4%.

13. Find the amount of $8645 from February 8, 1874, to September 8, 1875, at 8%.

14. Find the exact interest on $4000 from November 18, 1888, to August 24, 1892, at 5%.

15. Find the interest on $4140 from July 12, 1892, to May 12, 1895, at 9%.

16. Find the exact interest on $12,400 from June 15, 1889, to January 28, 1893, at 5%.

ANNUAL INTEREST

Annual Interest is simple interest on the principal and on any interest after it is due.

In order that annual interest may be collected, the note must contain the words, *interest payable annually.*

Find the amount of $840 for 4 yr. 5 mo. 15 da. at 5%, interest payable annually.

Principal	$840.00
Interest on principal for the whole time	187.25
“ “ $42 for 3 yr. 5 mo. 15 da. +	
“ “ $42 “ 2 yr. 5 mo. 15 da. +	
“ “ $42 “ 1 yr. 5 mo. 15 da. +	
“ “ $42 “ 5 mo. 15 da. =	
“ “ $42 “ 7 yr. 10 mo. =	16.45
Amount	$1043.70

In the above example, interest is computed on the principal and on the unpaid interest of 5%. It is important to remember, however, that interest on the unpaid interest should be computed at the *legal rate* in the state where the note is drawn, *unless the note explicitly states the rate of interest on the deferred payments.* Hence, if a note is given bearing interest at 4% payable annually, and the legal rate in the state is 6%, interest is computed on the principal at 4% and on the unpaid interest at 6%.

In the above example, if the legal rate is 6%, the amount due on the note would be as follows:

Principal	$840.00
Interest on principal for the whole time	187.25
Interest on $42 for 7 yr. 10 mo. at 6%	19.74

1. Find the amount of $4500 from June 14, 1887, to June 14, 1892, at 6%, interest payable annually.

2. Find the amount of $4000 from April 14, 1888, to October 14, 1892, at 5%, interest payable annually.

3. Find the amount of $1250 for 5 years 8 months 27 days at 4%, interest payable annually. Legal rate 6%.

4. Find the amount of $875 from June 1, 1886, to December 1, 1892, at 6%, interest payable annually.

5. Find the amount of $3500 for 5 years 3 months 18 days at 5%, interest payable annually. Legal rate 7%.

6. Find the amount of $5000 from April 1, 1884, to November 25, 1890, at 5%, interest payable annually.

7. Find the amount of $5875 for 4 years 7 months 18 days at 4%, interest payable annually. Legal rate 7%.

COMPOUND INTEREST

Compound Interest is interest on the principal and also on the interest, which at stated intervals is added to the principal. The interest may be compounded, or added to the principal, annually or semi-annually.

Find the amount of $875 for 3 years 8 months 24 days at 6%, interest compounded annually.

$875 × 1.06 = $927.50, amount for 1 year.
$927.50 × 1.06 = $983.15, amount for 2 years.
$983.15 × 1.06 = $1042.14, amount for 3 years.
$1042.14 × 1.044 = $1087.99, amount for whole time.

RULE. — *Find the amount of the principal for the first period. Treating this as a new principal, find the amount of it for the next period, and so on for the entire time.*

1. Find the amount of $2850 for 2 years 9 months 24 days at 6%, interest compounded annually.

2. Find the amount of $975 for 3 years 7 months 18 days at 5%, interest compounded annually.

3. Find the amount of $3500 for 1 year 9 months 12 days at 6%, interest compounded semi-annually.

4. Find the amount of $4500 for 2 years 5 months 24 days at 5%, interest compounded annually.

5. Find the amount of $3600 for 2 years 5 months 18 days at 8%, interest compounded semi-annually.

PROBLEM I IN INTEREST

To find the principal when the interest, time, and rate per annum are given:

What sum of money will yield $49 05 interest in 1 year 9 months 24 days at 6% ?

The interest on $1 for the given time at 6% is $.109. Since the whole interest is $49.05, there are as many dollars in the principal as there are $.109 in $49.05.

$49.05 ÷ $.109 = 450. Hence the principal is $450.

RULE.—*Divide the given interest by the interest on* $1 *for the given time at the given rate.*

1. On what sum will the interest at 6% for 1 year 5 months 24 days amount to $184.55 ?

2. Find the principal that will give $263.82 interest in 2 years 3 months 6 days at 4½%.

3. On what sum will the interest at 7% for 2 years 7 months 6 days amount to $268.35?

4. Find the principal that will produce $248.63 interest in 7 months 24 days at 5% simple interest.

PROBLEM II IN INTEREST

To find the principal when the amount, time, and rate per annum are given:

What sum of money will amount to $771.80 in 2 years 8 months 12 days at 5%?

The amount of $1 for the given time at 5% is $1.135. Since the whole amount is $771.80, there are as many dollars in the principal as there are $1.135 in $771.80.

$771.80 ÷ $1.135 = 680. Hence the principal is $680.

RULE. — *Divide the given amount by the amount of* $1 *for the given time at the given rate.*

1. What sum of money put at interest at 6% will amount to $1637.86 in 2 years 9 months 18 days?

2. Find the principal that will amount to $1426.74 in 2 years 3 months 18 days at 5%.

PROBLEM III IN INTEREST

To find the rate per annum when the principal, interest, and time are given:

At what rate per annum will $875 produce $168 interest in 3 years 2 months 12 days?

The interest on $875 for the given time at 1% is $28. Since the whole interest is $168, the rate is as many times 1% as $168 is times $28.

$168 ÷ $28 = 6. Hence the rate is 6%.

RULE.—*Divide the given interest by the interest on the principal for the given time at* 1%.

NOTE. — To find the interest at 1%, we generally find it at 6% and divide by 6. If the interest as computed at 6% is the same as the given interest, no further work is necessary, for the rate is evidently 6%. In all such examples, pupils should compare the interest at 6% with the given interest.

1. At what rate per annum will $7500 produce $450 interest in 1 year 6 months?

2. If I pay $63.50 for the use of $1500 for 8 months 14 days, what is the rate per annum?

3. Find the rate per annum when the amount of $1275 for 2 years 7 months 6 days is $1440.75.

4. Find the rate per annum when the interest on $1268 for 2 years 9 months 18 days is $266.28.

5. If a man pays $96.04 for the use of $840 for 1 year 7 months 18 days, what is the rate per annum?

PROBLEM IV IN INTEREST

To find the time when the principal, interest, and rate per annum are given:

In what time will $750 yield $58.50 interest at 6%?

The interest on $750 for 1 year at 6% is $45. Since the whole interest is $58.50, the time must be as many times 1 year as $58.50 is times $45.

$58.50 ÷ $45 = 1.3. Hence the time is 1.3 yr.

1.3 yr. = 1 yr. 3 mo. 18 da.

Rule. — *Divide the given interest by the interest on the principal for one year, and the quotient will be the number of years and fraction of a year.*

1. In what time will $1200 amount to $1246.33 at 5 per cent per annum?

2. In what time will $840 amount to $1275.05 at 5½ per cent per annum?

3. In what time will the interest on $486 amount to $18.71 at 6% per annum?

4. In what time will the interest on $950 amount to $216.49 at 7% per annum?

PARTIAL PAYMENTS

Partial Payments are payments in part of a note, bond, or other obligation.

An **Indorsement** is an acknowledgment of a payment on a note or bond, written on the back of the note or bond, stating the time and amount of the payment.

A note for $900, dated Apr. 1, 1892, bearing interest at 6%, was indorsed as follows: June 13, 1893, $85; Aug. 19, 1895, $90; Jan. 31, 1896, $250; Oct. 25, 1897, $150. How much was due on the note Sept. 13, 1899?

Principal		$900.00
Interest on $900 to June 13, 1893		64.80
Amount		$964.80
First payment, more than the interest		85.00
New principal		$879.80
Interest on $879.80 to Aug. 19, 1895		115.25
Second payment, less than the interest		
Interest on $879.80 to Jan. 31, 1896		23.76
Amount		$1018.81
Second payment	$90	
Third payment	250	340.00
New principal		$678.81
Interest on $678.81 to Oct. 25, 1897		70.60
Amount		$749.41
Fourth payment, more than the interest		150.00
New principal		$599.41
Interest on $599.41 to Sept. 13, 1899		67.73
Amount due		$667.14

This example is solved by the United States Rule, which is based on the following principles:

Each payment must be applied first to pay the interest due at the time the payment is made.

If the payment exceeds the interest then due, the excess is applied to reduce the principal.

If the payment is less than the interest then due, interest is computed on the same principal to the time when the sum of payments exceeds the interest then due.

RULE. — *Find the amount of the note to the time when the sum paid exceeds the interest then due.*

Subtract the sum paid from the amount, and treat the remainder as a new principal.

Proceed in the same manner with the new principal, and each succeeding one, to the time of settlement.

1. A note for $1800, dated Jan. 1, 1887, bearing interest at 6%, was indorsed as follows: Feb. 8, 1888, $260; Jan. 1, 1889, $375. What was due June 15, 1890?

2. A note for $1600, dated May 1, 1887, bearing interest at 5%, was indorsed as follows: Aug. 19, 1888, $285; Oct. 25, 1889, $125. How much was due Apr. 1, 1890?

3. A mortgage for $5400, dated May 4, 1884, was indorsed as follows: June 2, 1885, $275; Jan. 1, 1886, $578; Dec. 22, 1886, $1250; June 12, 1887, $868. How much was due on the mortgage May 4, 1889, with interest at 6%?

4. A note for $2800, dated Aug. 9, 1887, bearing interest at 4%, was indorsed as follows: Sept. 12, 1888, $225; Dec. 18, 1889, $75; Apr. 16, 1890, $348; Aug. 9, 1891, $295. Find the amount due on the note Aug. 9, 1892.

5. A note for $1750, dated Apr. 1, 1882, bearing interest at 5%, was indorsed as follows: Feb. 8, 1883, $245; Apr. 18, 1884, $50; Dec. 4, 1884, $362; Apr. 1, 1885, $475. How much was due on the note April 1, 1886?

COMMERCIAL DISCOUNT

Trade Discount, or **Commercial Discount,** is a deduction from the face of bills or the list price of goods.

The **Net Amount** of a bill is the face of the bill less the discount.

Sometimes bills of goods are sold subject to several discounts. In such cases, the first discount is computed on the face of the bill, the second on the sum left after the first discount is deducted, and so on.

1. What is the net amount of a bill of $865, if a discount of 25% and 5% is allowed?

2. What per cent of a bill does a merchant receive, if he gives a discount of 30% and 5%?

3. What is the net amount of a bill of $755, if the merchant gives discounts of 30%, 10%, and 5%?

4. A merchant accepted $493 in payment of a bill of $580. What per cent discount did he allow?

5. At a discount of $16\frac{2}{3}\%$, the net amount of a bill was $485. Find the amount of the discount.

6. How much more is a discount of 40% and 10% than a discount of 30% and 20% on a bill of $1386?

7. What is the net amount of a bill of $1400, if discounts of $\frac{1}{6}$ and 10% are allowed?

8. How much more is a discount of 30% than two discounts of 20% and 10% on a bill of $4650.50?

9. The net amount of a bill of books was $114.20, and the discount 20%. Find the list price of the books.

10. A merchant sold a bill of goods for $840 and gave a discount of $\frac{1}{4}$ and 10%. How much did he receive?

11. A bill of goods amounted to $750. The purchaser paid $625 for them. What per cent discount was allowed?

TRUE DISCOUNT

True Discount is a deduction made for the payment of a sum of money before it is due.

The **Present Worth** of a debt due at some future time is the sum which put at interest at the current rate will amount to the debt at the time it is due.

Find the present worth of a debt of $375 due in 8 months without interest, money being worth 6%.

$375 ÷ $1.04 = 360.58. Present worth is $360.58.

We are required to find what sum of money will amount to $375 in 8 months at 6% per annum. To find the present worth, we have therefore to find the principal when the amount, time, and rate per annum are given.

1. A merchant bought goods for $1270 on 4 months' credit. How much should have been discounted for payment at the time of purchase, money being worth 9%?

2. What is the present worth of a debt of $4500, $2300 of which is due in 8 months and the remainder in 1 year 2 months without interest, if discounted at 6%?

3. If a merchant buys flour for $835 on 4 months' credit, for how much cash must he sell it the same day to gain 15%, if discount is reckoned at 8%?

4. Find the present worth April 1, of two notes, one for $450 due Nov. 1, and the other for $500 due July 1, both without interest, if discounted at 6%.

5. A dealer bought apples for $720 on 2 months' credit and sold them the same day for $785 cash. He then paid for the apples, discounting at 6%. Find the profit.

6. Find the present worth of a debt of $4800, $\frac{1}{4}$ of which is due now, $1500 due in 6 months, and the remainder in 1 year, all without interest, if discounted at 8%.

BANK DISCOUNT

A promissory note is said to mature on the day when it is legally due. In most states, a note matures at the expiration of the time named in the note, except when that falls on Sunday or on a legal holiday, when it matures the day before or the day after, according to the law of the state where it is made. In some states, however, the maker is allowed three days, called **Days of Grace**, in which to pay the note after the time named has expired.

The **Maturity Value** of a note is the amount due on the note at maturity.

NOTE. — If a note does not bear interest, the maturity value is the face of the note. If a note bears interest, the maturity value is the amount of the note at maturity.

If A wishes to borrow money at a bank, he makes a note not bearing interest, payable at the bank to B. He then secures B's indorsement and presents the note at the bank. If the officers of the bank are satisfied that A and B are responsible persons, they will pay to A the maturity value of the note, less the interest on that sum from that day to the day of maturity, and retain the note in the bank. The note is then said to be discounted. When the note matures, A is expected to call at the bank and pay it.

NOTE. — Banks usually discount notes for short periods. The time is nearly always 30, 60, or 90 days.

The **Term of Bank Discount** is the number of days from the day of discount to the day of maturity.

The **Bank Discount** on a note is the interest on the maturity value for the term of discount.

The **Proceeds** of a note is the maturity value less the bank discount.

Banks sometimes discount notes that were not drawn for discount. The payee of a note may wish to obtain the money on it before it is due. In such a case, the payee indorses the note and presents it at a bank. If the note has but a short time to run, and if the officers of the bank are satisfied that the maker and payee of the note are responsible persons, they will discount it.

A note for $675, dated May 5, payable in 90 days without interest, was discounted at a bank July 16, at 6%. Find the proceeds, no grace being allowed.

Since the note does not bear interest, the maturity value of the note is $675.

Counting forward 90 days from May 5, we find that the note matures August 3.

The number of days from July 16, the day of discount, to August 3 is 18. Hence the term of discount is 18 days.

Since the interest on $675 for 18 days is $2.03, the bank discount is $2.03, and the proceeds $672.97.

A note for $850, dated March 8, payable in 4 months with interest at 6%, was discounted at a bank April 14 at 6%. Find the proceeds, no grace being allowed.

Since the note bears interest, the maturity value is the amount of $850 for 4 months at 6%, which is $867.

Since the note is given for 4 months, it matures July 8.

The number of days from April 14, the day of discount, to July 8 is 84 days. Hence the term of discount is 84 days.

Since the interest on $867 for 84 days is $12.14, the bank discount is $12.14, and the proceeds $854.86.

NOTE. — Since in most states days of grace are not allowed, all examples in this book are solved without days of grace.

1. Find the proceeds of a note for $475 payable in 90 days, if discounted at date at 6%.

2. Find the proceeds of a note for $735 payable in 60 days with interest at 8%, if discounted at 6%.

1. Find the proceeds of a note for $475 payable in 60 days, if discounted at date at 7%.

2. Find the proceeds of a note for $700 payable in 90 days, if discounted 36 days after date at 8%.

3. Find the bank discount and proceeds of a note for $1260, dated March 2, payable in 90 days without interest, if discounted March 14 at 6%.

4. Find the bank discount and proceeds of a note for $650, dated June 18, payable in 60 days with interest at 5%, if discounted June 27 at 7%.

5. Find the bank discount and proceeds of a note for $935, dated April 1, payable in 8 months without interest, if discounted July 18 at 8%.

6. Find the bank discount and proceeds of a note for $1600, dated April 1, 1901, payable in 4 months with interest at 5%, if discounted May 24, 1901, at 6%.

PROCEEDS GIVEN, TO FIND FACE

For what sum must a note be drawn for 90 days so that if discounted at date at 6%, the proceeds may be $394?

Since the interest on $1 for 90 days at 6% is $.015, the proceeds of $1 is $.985. Hence the face of the note is as many times $1 as $.985 is contained in $394.

$394.000 ÷ $.985 = 400. Hence the face is $400.

RULE. — *Divide the proceeds of the note by the proceeds of* $1 *for the term of bank discount.*

1. I wish to obtain $860 at a bank. For what sum must I give a note for 60 days to be discounted at 6%?

2. For what sum must a note be drawn for 90 days to obtain $825 at a bank, if discounted at date at 8%?

EXCHANGE

Exchange is the method of making payments in distant places by means of drafts.

A **Draft** is an order issued by one person to another to pay a sum of money to the order of a third person.

The **Face** of a draft is the sum named in the draft; the **Drawer**, or **Maker**, is the person who signs it; the **Drawee** is the one ordered to pay the money; the **Payee** is the one to whom the money is to be paid.

Exchange is generally conducted through banks; but when the draft is an order issued by one bank to another, this definition of a draft holds, for the cashier of the bank issuing it is the maker, and the cashier of the bank ordered to pay the money is the drawee.

When a draft is issued on a bank in New York, it is called a draft on New York; when it is issued on a bank in Chicago, it is called a draft on Chicago, etc.

Banks that wish to sell drafts on any important city have a deposit in some bank in that city; then all their drafts are issued on that bank. Nearly all banks throughout the country have a deposit in some bank in New York in order that they may sell drafts on New York.

A **Sight Draft** is a draft payable on presentation to the drawee. A **Time Draft** is a draft payable at a specified time after presentation, or after date.

The **Acceptance** of a time draft is the promise made by the drawee to pay the draft at maturity. This he does by writing the word *Accepted* across the face of the draft, with the date and his name. After a time draft has been accepted, it is then called an *acceptance*.

If Frank R. Page in Buffalo owes William H. King in New York, payment may be made by draft in two ways.

First, Page may go to a bank and buy a draft on New York and send it to King. In this case, the cashier of the bank in Buffalo is the maker, and the bank in New York is the drawee. Page may have the draft made payable to King, in which case he sends it to King without indorsement; or Page may have the draft made payable to himself, in which case he indorses it before he sends it to King.

$385.35 BUFFALO, N. Y., Nov. 23, 1901.

At sight pay to the order of William H. King Three hundred eighty-five and $\frac{35}{100}$ Dollars and charge to the account of the First National Bank of Buffalo.

To the Park National Bank, JAMES R. THOMPSON,
New York City *Cashier.*

If the draft is made payable to Page, it would differ from the above only in having Page's name in place of King's.

Second, King may *draw on* Page. In this case King is the maker of the draft and Page the drawee. King makes the draft payable to himself, or to the bank in New York where he has an account, or to some bank in Buffalo. King then indorses the draft and presents it at his bank. The bank sends the draft to a bank in Buffalo, and that bank notifies Page. If it is a sight draft, Page pays it at once. If it is a time draft, Page accepts it and pays it at maturity. If Page refuses to pay the draft or accept it, it is returned to the bank in New York.

$385.35 NEW YORK, NOV. 23, 1901.

At sight pay to the order of the First National Bank of Buffalo Three hundred eighty-five and $\frac{35}{100}$ Dollars and charge to the account of William H. King.

To John R. Page, WILLIAM H. KING.
205 Main St., Buffalo, N. Y.

When the banks in any city have not enough money on deposit in New York to meet the drafts they are issuing on New York, they must send money by express to meet them. This involves an expense, and exchange on New York is at a premium; that is, a draft will cost a certain per cent more than its face value.

When the banks in any city have on deposit in New York large sums on which they receive no interest, they will often sell drafts on New York at a discount so that they may get money to use without having it returned to them by express.

The cost of $1 of a draft is $1 plus the premium or $1 minus the discount, and this result diminished by the interest on $1 for the time named in a time draft.

How much will a sight draft for $875 cost, when exchange is at 1% premium?

$$\$1.01 \times 875 = \$883.75$$

Find the cost of a 30-day draft for $625, when exchange is at ½% discount and money is worth 6%.

Since exchange is at ½% discount, $1 of the draft costs $.995. The interest on $1 for 30 days at 6% is $.005. Hence $1 of the draft costs $.995 – $.005, or $.99.

$$\$.99 \times 625 = \$618.75$$

1. What is the cost of a sight draft for $875 when exchange is at ½% premium?

2. Find the cost of a 30-day draft for $640 when exchange is at 1½% premium and money is worth 6%.

1. Find the cost of a sight draft for $850 when exchange is at $\frac{3}{4}\%$ premium.

2. How large a sight draft can be bought for $550 when exchange is at $\frac{3}{4}\%$ discount?

3. Find the cost of a 30-day draft for $1725 when exchange is at 1% discount and money is worth 7%.

4. Find the cost of a 30-day draft for $575 when exchange is at $\frac{3}{4}\%$ premium and money is worth 6%.

5. When exchange is at $\frac{1}{2}\%$ premium and money is worth 6%, find the cost of a 60-day draft for $2675.

6. Find the face of a 30-day draft that costs $8000 when exchange is at $\frac{1}{4}\%$ discount and money is worth 7%.

STOCKS AND BONDS

When a very large sum of money is needed to establish and carry on any business, a number of persons combine to provide the necessary money. Such a combination of men, organized under the laws of some state, is a **Corporation**, or **Stock Company**. All the money contributed to start the business is the **Capital**, or **Capital Stock**, and the persons who invest the capital are the **Stockholders**.

The **Charter** of a corporation is the legal document obtained from the state specifying the rights and limitations of the corporation.

A **Share** is one of the equal parts into which the capital is divided. It is usually $100, $50, or $25.

A **Certificate of Stock** is a document given to each stockholder specifying the number of shares to which he is entitled and the original value of each share.

When a person buys or sells stock, he buys or sells a certificate, for it is the evidence of ownership.

The **Par Value,** or **Nominal Value,** of a share is its face value, or the sum named on its face.

When the business of a corporation is very profitable, the shares will sell *above par*, that is, for more than their nominal value; when the business is unprofitable, the shares will sell *below par.*

The **Market Value** of a stock is the price at which it will sell in market.

A **Dividend** is a sum of money divided among the stockholders from the net profits of the business.

Dividends are always computed on the par value of the stock without regard to its market value.

A **Surplus** is a portion of the net earnings left in the business. Most corporations accumulate a surplus.

When any government or corporation wishes to raise large sums of money, bonds are issued and sold. These are promissory notes, and they are secured by the property of the state, city, or corporation issuing them.

A **Mortgage** is a conditional sale of property, made to secure the payment of a bond.

We have seen that the dividend on the capital stock of any corporation may vary, because the net income varies. Governments and corporations cannot sell their bonds unless they specify the rate of interest. For this reason, bonds bear a fixed rate of interest.

A **Stock Broker** is a person who buys and sells stocks and bonds. **Brokerage** is the commission charged by a broker for buying or selling stocks for others, and in nearly all cases it is $\frac{1}{8}\%$ of the par value.

In all examples in stocks in this book, the par value of a share will be considered $100 unless otherwise stated.

When a man buys stock through a broker, it costs him the market value plus the brokerage; when he sells, he receives the market value less the brokerage.

Including brokerage at ⅛%, what does a share of stock cost at 94½%? At 115⅝? At 65⅜? At 138¾? At 48¼?

Find the cost of 80 shares of N.Y. Central R. R. stock at 145¾, brokerage ⅛%.

$$\$145.875 \times 80 = \$11,670.$$

How much will a man receive for 140 shares of D. L. & W. R. R. stock at 164⅝, brokerage ⅛%?

$$\$164.50 \times 140 = \$23,030.$$

1. Find the cost of 85 shares of Union Pacific R. R. stock at 8⅝% above par, brokerage ⅛%.

2. A man sold 35 shares of bank stock at 148⅞, brokerage ⅛%. How much did he receive for it?

3. Find the annual income on 125 shares of bank stock which pays a semi-annual dividend of 4%.

4. How much bank stock at 162½ can be bought for $7800? Find the dividend on it at 8%.

5. A man bought mining stock at 102⅞, brokerage ⅛%, paying $7725 for it. How many shares did he buy?

6. What is the annual income on 145 shares of railroad stock, if it pays a quarterly dividend of 1¼%?

7. A invested $3390 in stock at 28⅛, brokerage ⅛%, and sold it at 35, brokerage ⅛%. Find the gain.

8. B bought 85 shares of stock at 127⅜, brokerage ⅛%, and sold it at 135⅛, brokerage ⅛%. Find the gain.

9. A man sold through a broker 220 shares of stock at 165¾, brokerage ⅛%. How much did he receive for it?

A 4% stock is a stock that pays an annual dividend of 4%. A 5% bond is a bond that pays 5% interest.

Pupils must remember that the interest on bonds, like the dividend on capital stock, is computed on the par value.

1. A man invested $12,875 in 4% railroad stock at $102\frac{7}{8}$, brokerage $\frac{1}{8}$%. Find the annual dividend.

One share costs $103. $12,875 ÷ $103 = 125, the number of shares. At 4%, the dividend on one share is $4.

$$\$4 \times 125 = \$500.$$

2. What sum must be invested in 6% stock at $112\frac{3}{8}$, brokerage $\frac{1}{8}$%, to secure an annual income of $1260?

The dividend on one share is $6. $1260 ÷ $6 = 210, the number of shares that must be bought.

$$\$112.50 \times 210 = \$23,625.$$

1. A man invested $10,240 in 5% city bonds at 128. Find the annual income.

2. Find the annual income from an investment of $17,010 in 8% bank stock, bought at 162.

3. What sum must be invested in 6% stock at $138\frac{7}{8}$, brokerage $\frac{1}{8}$%, to secure an annual income of $900?

4. What sum is invested in 6% bonds at $3\frac{3}{8}$% premium to give a semi-annual income of $420?

5. A man bought stock at 95, paying $7125 for it. If it pays $4\frac{1}{2}$% semi-annually, what is the annual income?

6. A man receives $185 quarterly on 4% bonds, which he bought at $105\frac{5}{8}$, brokerage $\frac{1}{8}$%. Find the cost.

7. What sum must be invested in 5% bonds at $104\frac{5}{8}$, brokerage $\frac{1}{8}$%, to give quarterly interest of $187.50?

8. What sum must be invested in 7% bank stock at $115\frac{7}{8}$, brokerage $\frac{1}{8}$%, to give a semi-annual dividend of $770?

If a man buys 5% stock at 124⅞, brokerage ⅛%, what per cent does he receive on the investment?

A share of the stock costs $125. He must therefore invest $125 to get $5 income. In examples of this type we are required to find what per cent of the cost of a share is the income on a share.

Question: $5 is what per cent of $125?

1. What is the rate of income on money invested in 6% stock, if purchased at 84?

2. If a man buys 10% stock at 124⅞, brokerage ⅛%, what per cent does the investment pay?

3. What is the rate of income on money invested in 6% stock, if the stock is bought at 20% premium?

4. If stock that pays 1¾% quarterly is bought at 175, what rate of interest does it pay?

5. What sum must be invested in 5% railroad bonds at 105 to yield a quarterly income of $3000?

6. What will be received for some 8% bank stock which pays a semi-annual dividend of $420, if sold at 135?

7. A man sold 540 shares of 6% stock at 110, and invested the money in 4% bonds at 90. How much was his yearly income increased or diminished?

8. A man received $10,560 for railroad bonds, which a broker sold for him at 88⅛, brokerage ⅛%. If he bought the bonds at par, how much did he lose?

9. A man received $14,260 for 5% bonds sold at 115, and invested the money in 6% bank stock at 124. How much was his annual income increased or diminished?

10. A young man inherited $43,200. He invested half of it in 5% railroad stock at 96, and the other half in 6% bank stock at 112½. What was his yearly income?

PARTNERSHIP

A **Partnership** is an association of two or more persons for the transaction of business. The **Partners** are the persons associated in business.

The **Capital** is the money or property invested in the business. The **Resources,** or **Assets,** of a firm are its property of all kinds, together with the amounts due it. The **Liabilities** of a firm are its debts.

PRINCIPLE. — *The gains and losses of a firm are shared in proportion to the amount of capital each partner invests, and the length of time it is in the business.*

To find the partners' gain or loss when their capital is employed for equal periods of time:

A and B entered into partnership. A furnished $6000 of the capital, and B $4000. At the end of a year their profits were $2600. Find each one's share.

$6000 + $4000 = $10,000, the whole capital.

A has $\frac{6000}{10000}$ of the capital, or $\frac{3}{5}$. $\frac{3}{5}$ of $2600 = $1560.

B has $\frac{4000}{10000}$ of the capital, or $\frac{2}{5}$. $\frac{2}{5}$ of $2600 = $1040.

1. A, B, and C engage in business with a joint capital of $18,000. A put in $9000; B, $6000; and C, the remainder. Their profits were $4500. Find C's share.

2. A and B bought land for $36,000, A investing $22,000 and B the remainder. If they sold it at a profit of $7200, what was each one's share of the gain?

3. A, B, and C formed a partnership with a joint capital of $22,000. A and B each put in $7000 and C the remainder. They gained $7062. Find C's share.

To find the partners' gain or loss when their capital is employed for unequal periods of time:

A and B formed a partnership for a year, each investing \$6000. After 3 months A put in \$3000 more, and at the end of 6 months B put in \$3000 more. Their profits were \$9450. Find each partner's share.

$$\text{A}\left\{\begin{array}{l}\$6000 \times 12 = \$72{,}000 \\ \$3000 \times \ \ 9 = \underline{\$27{,}000} \\ \qquad\qquad\quad\ \ \$99{,}000\end{array}\right. \qquad \text{B}\left\{\begin{array}{l}\$6000 \times 12 = \$72{,}000 \\ \$3000 \times \ \ 6 = \underline{\$18{,}000} \\ \qquad\qquad\quad\ \ \$90{,}000\end{array}\right.$$

A has \$6000 for 12 months and \$3000 for 9 months, all of which is equivalent to \$99,000 for 1 month. B has \$6000 for 12 months and \$3000 for 6 months, all of which is equivalent to \$90,000 for 1 month. The whole capital for 1 month is \$189,000.

A has $\frac{99000}{189000}$ of the capital, or $\frac{99}{189}$. $\frac{99}{189}$ of \$9450 = \$4950.

B has $\frac{90000}{189000}$ of the capital, or $\frac{90}{189}$. $\frac{90}{189}$ of \$9450 = \$4500.

1. A, B, and C bought property, A and B each investing \$24,000 and C \$12,000. They soon sold it at a profit of \$18,000. To how much of the profit was each entitled?

2. A, B, and C entered into partnership. A put in \$8000 for 1 year; B, \$6000 for 9 months; and C, \$5000 for 6 months. The profits were \$6450. Find C's share.

3. Jan. 1, A and B entered into partnership for one year. A put in \$6000 and B \$10,000. A put in \$2000 more Apr. 1, and B took out \$1500 May 1. Their profits for the year were \$7750. What was each partner's share?

4. A started in business May 1, 1900. July 1, he formed a partnership with B, and Sept. 1, C became a member of the firm. May 1, 1901, A was entitled to \$1680 of the profits; B, \$1120; and C, \$1568. If the whole capital was \$16,000, how much did each invest?

1. A, B, and C engage in business together. A put in $8000; B, $2000 more than A; and C, $4000 more than B. Their profits were $6432. Find each one's share.

2. A, B, and C formed a partnership with a capital of $25,000. A and B each put in twice as much as C. If they gained $6425, what was each man's share?

3. A and B formed a partnership for a year, each investing $6000. After 3 months A put in $2000 more, and at the end of 6 months B put in $3000 more. If their profits were $8883, what was each man's share?

4. Apr. 1, A and B entered into partnership for one year, each furnishing $4000 capital. July 1, A added $1000, and Oct. 1, he added $2000 more. June 1, B added $4000, and Nov. 1, he took out $2000. Their profits for the year were $7448. How much should each receive?

5. Jan. 1, A started in business with $6000. Apr. 1, he took B into partnership. Later they took C as partner with $10,000 capital. At the end of the year their profits were $3916, of which A took $1320 and C $550. Find B's capital and the time when C entered the firm.

6. A, B, and C engaged in partnership, each investing $10,000. After 3 months A added $2000 to his capital. At the end of 4 months B took out $2000, and C increased his capital to $15,000. Their profits at the end of the year were $8442. What was each partner's share?

7. Oct. 1, A and B formed a partnership. A put in $8000, and B $10,000. Jan. 1, A added $4000, and Apr. 1, he took out $1000. Jan. 1, B took out $2000, and Apr. 1, he added $3000. Apr. 1, they took in C as partner, who put in $8000. Their profits for the year ending Oct. 1 amounted to $6958. What was each partner's share?

RATIO AND PROPORTION

Ratio is the quotient obtained by dividing one number by another of the same kind.

The ratio of 15 to 5 is 3. The ratio of 5 to 35 is $\frac{1}{7}$.

The **Terms** of a ratio are the two numbers compared.

The **Antecedent** is the first term of a ratio. The **Consequent** is the second term of a ratio.

Since a ratio is determined by division, a ratio may be indicated by the sign of division, or by writing the dividend above the divisor in the form of a fraction. It is also indicated by the colon.

$14 \div 3$ $\frac{14}{3}$ $14 : 3$

1. 45 : 15. 12 : 48. 45 : 30. 36 : 48.
2. 15 : 75. 70 : 35. 30 : 45. 30 : 12.
3. 64 : 16. 25 : 75. 70 : 20. 48 : 60.

When two fractions have a common denominator, the ratio of one to the other is the same as the ratio of their numerators.

1. $\frac{2}{5} : \frac{4}{5}$. $\frac{1}{3} : \frac{1}{9}$. $\frac{1}{3} : \frac{1}{2}$. $\frac{3}{4} : \frac{1}{2}$.
2. $\frac{2}{7} : \frac{6}{7}$. $\frac{3}{4} : \frac{3}{8}$. $\frac{1}{2} : \frac{2}{3}$. $\frac{3}{8} : \frac{1}{4}$.
3. $\frac{3}{8} : \frac{3}{4}$. $\frac{2}{3} : \frac{1}{9}$. $\frac{2}{9} : \frac{1}{3}$. $\frac{1}{2} : \frac{3}{8}$.
4. $\frac{1}{9} : \frac{2}{3}$. $\frac{3}{4} : \frac{1}{8}$. $\frac{3}{8} : \frac{1}{2}$. $\frac{5}{8} : \frac{1}{2}$.
5. 2 ft. : 8 in. \$3 : \$.50. 2 yd. : 2 ft.
6. 2 ft. : 4 yd. \$4 : \$.80. 4 ft. : 3 yd.
7. 2 ft. : 3 in. \$3 : \$.25. 4 yd. : 8 ft.
8. 9 in. : 3 ft. \$6 : \$.75. 11 ft. : 2 rd.

A **Proportion** is an expression of equality between two ratios. The equality is indicated in two ways.

$9 : 3 = 6 : 2$ $\qquad$ $9 : 3 :: 6 : 2$

The **Terms** of a proportion are the four terms of the two equal ratios.

The **Extremes** are the first and fourth terms of a proportion. The **Means** are the second and third terms.

A **Simple Proportion** is a proportion whose terms are expressed by single numbers.

Example given above.

A **Compound Proportion** is a proportion, one or both of whose terms are expressed by their factors.

$8 : 4 :: 45 : 15$ $\qquad$ $12 : 8 :: 15 : 20$

$9 : 6$ $\qquad$ $10 : 5 :: 24 : 6$

If three terms of any proportion are known, the other term may be found. Remember the ratios are equal.

1. $60 : 15 :: x : 12$ $\qquad$ 2. $x : 45 :: 25 : 75$.

Since 60 in the first example is 4 times 15, the second antecedent is 4 times 12, or 48. Since 25 in the second example is $\frac{1}{3}$ of 75, the first antecedent is $\frac{1}{3}$ of 45, or 15.

1. $13 : 65 :: 14 : x$. $\qquad$ 2. $84 : x :: 77 : 11$.

Since 65 in the first example is 5 times 13, the second consequent is 5 times 14, or 70. Since 11 in the second example is $\frac{1}{7}$ of 77, the first consequent is $\frac{1}{7}$ of 84, or 12.

1. $8 : 4 :: 7 : x$. $\qquad$ $x : 5 :: 9 : 6$. $\qquad$ $15 : x :: 9 : 12$.
2. $4 : 6 :: x : 8$. $\qquad$ $5 : x :: 6 : 2$. $\qquad$ $12 : 48 :: 7 : x$.
3. $x : 4 :: 6 : 9$. $\qquad$ $6 : 8 :: x : 9$. $\qquad$ $7 : x :: 20 : 30$.
4. $7 : 3 :: 9 : x$. $\qquad$ $x : 8 :: 4 : 3$. $\qquad$ $15 : 90 :: x : 4$.

PRINCIPLE. — *In any proportion the product of the means is equal to the product of the extremes.*

This principle enables us to find the missing term in any proportion without knowing the ratio.

$6:4=x:6$	$x:8=3:4$	$14:x=7:4$
$4x=36$	$4x=24$	$7x=56$
$x=9$	$x=6$	$x=8$

There are many problems in arithmetic in which the ratio of two given numbers is equal to the ratio of two other numbers, one of which is to be found.

To solve a problem by proportion, the pupil must first determine what two pairs of numbers in the problem have equal ratios. He then expresses the equality of these ratios in the form of a proportion, denoting the number to be found by x, and finds the missing term.

NOTE. — It must be remembered that the terms of a ratio must be expressed in the same unit.

At the same price, two quantities of anything have the same ratio as their respective cost.

If 48 yards of silk cost $84, how much will 25 yards cost at the same price per yard?

$$48:25::84:x$$

1. If 5 yards of lace cost $8.75, how many yards will cost $28 at the same price per yard?

2. If 9 dozen pencils cost $1.80, how many gross can be bought for $30 at the same price?

3. If a piece of oil cloth 5 ft. long and 4 ft. wide costs $1.60, how many square yards will cost $9?

4. If 70 sq. rd. of ground yield 44 bu. of potatoes, how many bushels will 3½ acres yield at the same rate?

5. If 5 yd. 1 ft. 6 in. of ribbon cost $3.74, how many yards can be bought for $5.61?

6. How many bushels of potatoes will cost $6.38, when 2 bushels 3 pecks cost $2.42?

7. If the interest on $850 is $29.75, what principal will yield $140 interest in the same time?

8. If 15 quarts of berries cost $1.80, how many crates of 32 quarts each will cost $24?

9. If 2 lb. 9 oz. of tea cost $1.64, how much will 5 lb. 14 oz. cost at the same price per pound?

10. If a train runs 255 miles in 5 hr. 40 min., how far will it run in 9 hr. 20 min. at the same rate?

11. How much will 28 yards of silk cost at the same price per yard at which 17 yards cost $23.80?

The solution of some problems involves a compound proportion. In such examples one or more of the terms are expressed by their factors.

If 8 men can build 108 rods of wall in 18 days, in how many days can 12 men build 72 rods?

The numbers of days' work have the same ratio as the numbers of rods of wall built. The number of days' work required to build 108 rods is the product of 8 and 18, and the number of days' work required to build 72 rods is the product of 12 and x.

$$\begin{array}{l} 8 : 12 :: 108 : 72 \\ 18 : x \end{array}$$

$$(12 \times 108)x = 8 \times 18 \times 72$$

$$x = 8$$

NOTE. — The solution is shortened by cancelling equal factors on opposite sides of the sign of equality.

1. If 8 men can do $\frac{2}{3}$ of a piece of work in 9 days, how many men can do the whole of it in 4 days?

2. If 864 cubic inches of water weigh 31 pounds, how many pounds will a cubic yard weigh?

3. If 9 men can do a piece of work in 12 days, how many men can do 4 times as much in 18 days?

4. If 25 yards of carpet cost $43.75, how many yards will cost $77 at the same price per yard?

5. In how many days can 16 men do $\frac{3}{5}$ of a piece of work, if 20 men can do $\frac{7}{8}$ of it in 14 days?

6. How many men can build 123 rods of wall in $20\frac{1}{2}$ days, if 5 men can build it in 16 days?

7. If a farmer receives $7.37 for .44 of a ton of hay, what will he receive for $4\frac{1}{5}$ tons?

8. In how many days can 24 men do $\frac{4}{5}$ of a piece of work, if 15 men can do the whole of it in 40 days?

9. How many bushels of oats will 11 horses eat in 13 days, if 17 horses eat 55 bu. 3 pk. 1 qt. in 21 days?

10. How many hours a day must 4 men work to do half as much work in 10 days as 15 men can do in 4 days when they work 10 days a day?

11. Twenty-four men are engaged to do a piece of work in 15 days. After 5 days 8 men stop work. In how many days can the rest of the men complete the work?

12. How high must a pile of wood be that is 80 feet long and 4 feet wide to be worth $71.25, if a pile 64 feet long, 8 feet wide, and 8 feet high is worth $152?

13. If 14 men can dig a ditch 120 ft. long, 8 ft. wide, and 6 ft. deep in 12 days, in how many days can 18 men dig a ditch 180 ft. long, 12 ft. wide, and 4 ft. deep?

14. Find the ratio of a field 35 rods long and 18 rods wide to a field 21 rods long and 15 rods wide.

15. If 6 men earn $150 in 20 days, how many men will earn $78.75 in 9 days at the same rate?

16. How many miles can 48 tons be carried for the money paid for carrying 86 tons 192 miles?

17. How many men can do $\frac{5}{8}$ of a piece of work in 10 days, if 8 men can do the whole of it in 20 days?

18. How many days will it take 15 men to cut 405 cords of wood, if 13 men can cut 780 cords in 40 days?

19. If 14 ounces of tea cost 70 cents, how much will $4\frac{3}{4}$ pounds cost at the same price per pound?

20. On what sum will the interest amount to $54, if the interest on $640 for the same time is $28.80?

21. If 12 boys or 8 men can do a piece of work in 20 days, in how many days can 25 boys and 10 men do it?

22. If 32 men can do a piece of work in 45 days, in how many days can they complete the same work with the assistance of 8 more men?

23. Eighteen men are engaged to do a piece of work in 24 days. After 6 days 6 men stop work. In how many days can the rest of the men complete the work?

24. If 12 men can build a wall 48 yd. long, 5 ft. wide, and 8 ft. high in 10 days of 8 hours each, how many men will it take to build a wall 120 ft. long, 3 ft. wide, and 12 ft. high in 5 days of 9 hours each?

25. If 7 men can excavate a cellar 50 ft. long, 36 ft. wide, and 8 ft. deep in 20 days of 8 hours each, how many hours a day must 12 men work to excavate a cellar 45 ft. long, 25 ft. wide, and 6 ft. deep in 5 days?

INVOLUTION

A **Power** of a number is the product obtained by taking the number two or more times as a factor.

The powers of a number are named from the number of times it is taken as a factor.

The **Square, or Second Power,** of a number is the product obtained by taking it twice as a factor.

The **Cube, or Third Power,** of a number is the product obtained by taking it three times as a factor.

An **Exponent** is a small figure written at the right of a number, and a little above it, to denote the power.

$8^2 = 8 \times 8.$ $5^3 = 5 \times 5 \times 5.$ $3^4 = 3 \times 3 \times 3 \times 3.$

Involution is the process of finding a power of a number.

1. 28^2. 2. 47^3. 3. 58^2. 4. 15^4.
5. 65^3. 6. 87^2. 7. 74^3. 8. 24^4.

A fraction is raised to a power by raising each term of the fraction to the required power.

1. $(\frac{5}{8})^3$. 2. $(\frac{12}{17})^2$. 3. $(8\frac{1}{2})^2$. 4. $(1.5)^3$.
5. $(\frac{7}{9})^3$. 6. $(\frac{16}{25})^2$. 7. $(3\frac{2}{3})^3$. 8. $(5.6)^4$.

EVOLUTION

A **Root** of a number is one of the equal factors of it.

The **Square Root** of a number is one of the two equal factors of the number.

4 is the square root of 16. 7 is the square root of 49.

The **Cube Root** of a number is one of the three equal factors of the number.

3 is the cube root of 27. 5 is the cube root of 125.

Evolution is the process of finding a root of a number.

As a knowledge of square root is of little value and but little applied in arithmetic, we shall simply illustrate the process without attempting any explanation.

	21′06′81	459		57′91′21	761		41′99′04	648
	16			49			36	
85	506		146	891		124	599	
	425			876			496	
909	8181		1521	1521		1288	10304	
	8181			1521			10304	

RULE. — *Separate the number into periods of two figures each, beginning at the right.*

Find the greatest square in the left-hand period, and write its square root for the first figure of the root.

Subtract this square from the left-hand period, and to the remainder annex the next period for a dividend.

Double the root already found for a partial divisor. Divide the dividend, omitting the right-hand figure, by the partial divisor, and annex the quotient to the root and also to the divisor. Multiply the completed divisor by the last figure of the root, subtract the product from the dividend, and to the remainder annex the next period for a new dividend, continuing the process until all the periods have been used.

NOTE. — If any partial divisor is not contained in the dividend, annex a cipher to the root and also to the partial divisor, and annex the next period of the number to the dividend.

Find the square root of the following numbers:

1. 18769.
2. 811801.
3. 13734436.
4. 24403600.
5. 94249.
6. 258064.
7. 33860761.
8. 49984900.
9. 212521.
10. 396900.
11. 16224784.
12. 35892081.

SQUARE ROOT OF A DECIMAL

RULE. — *Separate the decimal into periods of two figures each, beginning at the decimal point.*

The process is the same as with whole numbers.

From the right of the root point off as many decimal places as there are periods of decimal places.

NOTE. — If we wish to obtain the square root of a decimal to 2 decimal places, we should have 4 decimal places in the number; if we wish to carry the work to 3 decimal places, we should have 6 decimal places in the number; and so on. The number of decimal places may be increased by annexing ciphers.

$\sqrt{1322}$ indicates the square root of 1322.

1. $\sqrt{1.008}$. 2. $\sqrt{.4}$. 3. $\sqrt{.009}$. 4. $\sqrt{904.2}$.
5. $\sqrt{64.05}$. 6. $\sqrt{.7}$. 7. $\sqrt{.025}$. 8. $\sqrt{80.07}$.
9. $\sqrt{138.7}$. 10. $\sqrt{.9}$. 11. $\sqrt{1.01}$. 12. $\sqrt{3.504}$.

SQUARE ROOT OF A COMMON FRACTION

RULE. — *If both terms of the fraction are squares, find the square root of each term separately.*

If either term is not a square, reduce the fraction to a decimal and find the square root of the decimal.

1. $\sqrt{5\frac{1}{2}}$. 2. $\sqrt{\frac{5}{6}}$. 3. $\sqrt{4\frac{2}{3}}$. 4. $\sqrt{6\frac{1}{4}}$.
5. $\sqrt{8\frac{3}{4}}$. 6. $\sqrt{\frac{4}{7}}$. 7. $\sqrt{2\frac{7}{9}}$. 8. $\sqrt{5\frac{2}{7}}$.
9. $\sqrt{7\frac{1}{9}}$. 10. $\sqrt{\frac{7}{9}}$. 11. $\sqrt{7\frac{4}{5}}$. 12. $\sqrt{9\frac{5}{6}}$.

CUBE ROOT

The two examples on the next page illustrate the process of finding the cube root of a number.

$$\begin{array}{r l l}
 & 34'328'125 & \underline{\lvert 325} \\
 & 27 & \\ \cline{2-2}
2700 & 7328 & \\
180 & & \\
4 & & \\ \cline{1-1}
2884 & 5768 & \\ \cline{2-2}
307200 & 1560125 & \\
4800 & & \\
25 & & \\ \cline{1-1}
312025 & 1560125 & \\ \cline{2-2}
\end{array}
\qquad
\begin{array}{r l l}
 & 201'230'056 & \underline{\lvert 586} \\
 & 125 & \\ \cline{2-2}
7500 & 76230 & \\
1200 & & \\
64 & & \\ \cline{1-1}
8764 & 70112 & \\ \cline{2-2}
1009200 & 6118056 & \\
10440 & & \\
36 & & \\ \cline{1-1}
1019676 & 6118056 & \\ \cline{2-2}
\end{array}$$

RULE. — *Separate the number into periods of three figures each, beginning with units.*

Find the greatest cube in the left-hand period, and write its cube root for the first figure of the root.

Subtract this cube from the left-hand period, and to the remainder annex the next period for a dividend.

Square the root already found, multiply the square by 3, *and to the product annex two ciphers for a partial divisor.*

Divide the dividend by the partial divisor, and annex the quotient to the root.

Multiply the first part of the root by the second, multiply that product by 3, *and to the result annex one cipher. Add this number and the square of the second part of the root to the partial divisor for the complete divisor.*

Multiply the complete divisor by the second part of the root, and subtract the product from the dividend.

Proceed in the same manner with all the periods.

NOTE. — If any partial divisor is not contained in the dividend, annex one cipher to the root, annex two ciphers to the partial divisor, and annex the next period to the dividend.

$\sqrt[3]{2354}$ indicates the cube root of 2354.

1. $\sqrt[3]{131872229}$. **2.** $\sqrt[3]{14348907}$. **3.** $\sqrt[3]{201230056}$.

4. $\sqrt[3]{303464448}$. **5.** $\sqrt[3]{27270901}$. **6.** $\sqrt[3]{315821241}$.

7. $\sqrt[3]{354894912}$. **8.** $\sqrt[3]{43614208}$. **9.** $\sqrt[3]{521660125}$.

CUBE ROOT OF A DECIMAL

RULE. — *Separate the decimal into periods of three figures each, beginning at the decimal point.*

The process is the same as with whole numbers.

From the right of the root point off as many decimal places as there are periods of decimal places.

NOTE. — If we wish to obtain the cube root of a decimal to 2 decimal places, we should have 6 decimal places in the number; if we wish to carry the work to 3 decimal places, we should have 9 decimal places in the number; and so on. The number of decimal places may be increased by annexing ciphers.

1. $\sqrt[3]{9.1819}$. **2.** $\sqrt[3]{4.3953}$. **3.** $\sqrt[3]{.27}$. **4.** $\sqrt[3]{401947272}$.

5. $\sqrt[3]{55.132}$. **6.** $\sqrt[3]{66.973}$. **7.** $\sqrt[3]{.64}$. **8.** $\sqrt[3]{714516984}$.

9. $\sqrt[3]{971.19}$. **10.** $\sqrt[3]{205.59}$. **11.** $\sqrt[3]{.47}$. **12.** $\sqrt[3]{743677416}$.

LONGITUDE AND TIME

A **Meridian** is an imaginary line on the earth passing from pole to pole.

Longitude is distance east or west from a given meridian. The meridian from which longitude is usually reckoned is that which passes through Greenwich, England.

Longitude is measured in degrees, minutes, and seconds. The circumference of the earth is divided into 360 degrees, a degree into 60 minutes, and a minute into 60 seconds.

The longitude of a place is the number of degrees the place is east or west of the meridian of Greenwich, and the difference in longitude between two places is the number of degrees from one place to the other.

If two places are both east or both west, the difference in longitude is found by subtracting the less from the greater; if one place is east and another west, the difference in longitude is found by addition.

1. A is 35° 45′ 50″ east, and B is 85° 12′ 30″ east. Find the difference in longitude.

2. A is 38° 25′ 35″ east, and B is 72° 42′ 48″ west. Find the difference in longitude.

3. A is 120° 15′ 12″ west and B is 45° 55′ 40″ west. Find the difference in longitude.

The relative position of the sun to any meridian determines the time of all places on that meridian. When the rays of the sun are vertical at any point of a meridian, it is noon at all places on that meridian. Since the sun appears to move around the earth from east to west, when it is noon on any meridian, it is past noon at all places east of it, and before noon at all places west of it.

To find the time at one place when the time at another place and the difference in time are known:

RULE. — *To the time of the westerly place add the difference in time; or from the time of the easterly place subtract the difference in time.*

NOTE. — In this work call 1 o'clock P.M. the 13th hour, 2 o'clock the 14th hour, 3 o'clock the 15th hour, and so on.

1. The difference in time between two places is 3 hr. 25 min. When it is 30 min. past 1 P.M. at the easterly place, what is the time at the westerly place?

2. The difference in time between two places is 2 hr. 40 min. When it is 15 min. past 11 A.M. at the westerly place, what is the time at the easterly place?

3. The difference in time between two places is 5 hr. 45 min. When it is 45 min. past 3 P.M. at the easterly place, what is the time at the westerly place?

The sun appears to move around the earth in 24 hours, hence it appears to move through 360° in 24 hours, or 15° in 1 hour, 15′ in 1 minute, 15″ in 1 second.

To find the difference in longitude between two places when the difference in time is given or can be found:

RULE. — *Multiply the difference in time, expressed in hours, minutes, and seconds by* 15, *and the respective products will denote the number of degrees, minutes, and seconds in the difference in longitude.*

1. The difference in time between two places is 5 hr. 4 min. Find the difference in longitude.

2. The difference in time between two places is 2 hr. 15 min. 12 sec. Find the difference in longitude.

3. Find the longitude of a place whose time is 20 min. 40 sec. past 8 P.M. when it is noon at Greenwich.

4. The difference in time between two places is 6 hr. 8 min. 20 sec. Find the difference in longitude.

5. When it is 11 o'clock A.M. at New York, it is 5 min. 21 sec. past 4 P.M. at Paris. The longitude of New York is 74° 3″ west. What is the longitude of Paris?

To find the difference in time between two places when the difference in longitude is given:

RULE. — *Divide the difference in longitude, expressed in degrees, minutes, and seconds by* 15, *and the respective quotients will denote the number of hours, minutes, and seconds in the difference in time.*

1. The difference in longitude between two places is 65° 20′ 15″. What is the difference in time?

2. The difference in longitude between two places is 87° 45′. What is the difference in time?

3. The difference in longitude between two places is 72° 18′ 24″. What is the difference in time?

4. The difference in time between two places is 9 hr. 45 min. Find the difference in longitude.

5. The difference in longitude between two places is 58° 20′ 45″. Find the difference in time.

6. The longitude of Philadelphia is 75° 10′ west and that of St. Paul 93° 4′ 55″ west. When it is 15 min. past 1 P.M. at Philadelphia, what is the time at St. Paul?

7. The difference in time between two places is 4 hr. 10 min. When it is 10 o'clock A.M. at the westerly place, what is the time at the easterly place?

8. When it is noon at Boston, it is 34 min. 29 sec. past 8 A.M. at San Francisco. The longitude of Boston is 71° 3′ 30″ west. What is the longitude of San Francisco?

9. The difference in longitude between two places is 62° 15′ 40″. When it is 25 min. past 10 A.M. at the westerly place, what is the time at the easterly place?

10. The longitude of Philadelphia is 75° 10′ west. When it is 10 o'clock A.M. at Philadelphia, it is 10 min. 1 sec. past 3 P.M. at Paris. Find the longitude of Paris.

11. The difference in time between two places is 8 hr. 10 min. 40 sec. Find the difference in longitude.

12. What is the difference in time between Greenwich and a place 120° 40′ 15″ west of it?

13. What is the longitude of a place whose time is 40 min. 20 sec. past 3 P.M., when it is noon at Greenwich?

14. The longitude of Canton is 113° 14′ east. When it is 10.45 A.M. at Greenwich, what is the time at Canton?

15. What is the difference in time between Greenwich and a place 95° 16′ east of it?

16. The difference in time between two places is 10 hr. 15 min. When it is 35 min. past 4 P.M. at the easterly place, what is the time at the other place?

17. The longitude of New York is 74° 3″ west and that of Paris 2° 20′ 15″ east. When it is 20 min. 40 sec. past 9 A.M. at New York, what is the time at Paris?

18. The longitude of Boston is 71° 3′ 30″ west and that of San Francisco 122° 26′ 15″ west. When it is noon at Boston, what is the time at San Francisco?

19. The longitude of Albany is 73° 44′ 53″ west and that of St. Paul is 93° 4′ 55″ west. How much earlier does the sun rise at Albany than at St. Paul?

20. The difference in longitude between two places is 84° 16′ 20″. When it is 40 min. 20 sec. past 3 P.M. at the easterly place, what is the time at the other place?

21. The longitude of Cincinnati is 84° 26′ west and that of Berlin is 13° 23′ 45″ east. When it is 20 min. 30 sec. past 9 A.M. at Cincinnati, what is the time at Berlin?

22. When it is 30 min. past 2 P.M. at Paris, it is 40 min. 47 sec. past 8 A.M. at Chicago. The longitude of Paris is 2° 20′ 15″ east. Find the longitude of Chicago.

METRIC SYSTEM

The **Metric System** is a system of weights and measures whose units increase and decrease on the decimal scale. It is called the metric system, because the unit is the meter, all the units of length, surface, volume, capacity, and weight being derived from the meter.

LINEAR MEASURE

10 millimeters	mm	= 1 centimeter	cm
10 centimeters		= 1 decimeter	dm
10 decimeters		= 1 meter	m
10 meters		= 1 dekameter	Dm
10 dekameters		= 1 hektometer	Hm
10 hektometers		= 1 kilometer	Km

The unit of linear measure is the meter, and all the other units are named by attaching certain prefixes to the name of the unit. *Milli* means thousandth, *centi* means hundredth, *deci* means tenth, *deka* means 10, *hekto* means 100, *kilo* means 1000. Hence a millimeter is 1 thousandth of a meter, a centimeter is 1 hundredth of a meter, a decimeter is 1 tenth of a meter, a dekameter is 10 meters, a hektometer is 100 meters, a kilometer is 1000 meters.

It will thus be seen that each unit in linear measure is 1 tenth of the next higher unit and 10 of the next lower. Hence, in reducing any number of linear units to the next lower, we multiply by 10; in reducing any number of units to the next higher, we divide by 10.

NOTE. — The pupil will remember that in multiplying by 10, he moves the decimal point one place to the right; in dividing by 10, he moves the decimal point one place to the left.

$$3657^{\mathrm{cm}} = 365.7^{\mathrm{dm}} = 36.57^{\mathrm{m}} = 3.657^{\mathrm{Dm}} = .3657^{\mathrm{Hm}}$$

1. Express 46395^{cm} as meters; as hektometers; as decimeters; as kilometers; as dekameters.

2. Express 34^{Km} as meters; as centimeters; as dekameters; as decimeters; as hektometers.

3. Add 9300^{dm}, 7368^{m}, 4826^{Dm}, 495^{Hm}, and 725^{Km}, and express the sum as kilometers.

4. Add 8400^{cm}, 6200^{dm}, 4835^{m}, 1265^{Dm}, 475^{Hm}, and 85^{Km}, and express the sum as hektometers.

5. At 8 cents a meter, what will it cost to enclose a rectangular field 14^{Dm} 6^{m} long and 8^{Dm} 8^{m} wide?

6. A merchant sold 8^{m} 4^{dm} of ribbon to each of 7 ladies. What did he get for it at 4 cents a decimeter?

7. How many rails 9^{m} 4^{dm} long will it take to build a double track railway 5^{Km} 6^{Hm} 4^{Dm} long?

8. Using boards 4^{m} 6^{dm} long, how many boards will it take to build a fence 4 boards high 9^{Hm} 2^{Dm} long?

SURFACE MEASUREMENTS

The units of surface measurements are squares whose dimensions are the corresponding units of length. The names of the units are the same as those used in linear measure with the word *square* prefixed. A square meter is therefore a square each of whose sides is 1 meter. Since a square meter is 10 decimeters long and 10 decimeters wide, it contains 100 square decimeters. Thus each unit of surface is 1 hundredth of the next higher and 100 of the next lower. Hence, in reducing any number of square units to the next lower, we multiply by 100; in reducing any number of units to the next higher, we divide by 100.

$$365275^{\mathrm{sq\,dm}} = 3652.75^{\mathrm{m}} = 36.5275^{\mathrm{Dm}} = .365275^{\mathrm{Hm}}$$

In measuring land, the square meter is called a *centare* ^{ca}, the square dekameter is called an *are* ^{a}, and the square hektometer is called a *hectare* ^{Ha}.

1. How many square decimeters are there in a piece of paper 24^{m} long and 1^{m} 4^{dm} wide?

2. How many hectares are there in a rectangular field 4^{Hm} 2^{Dm} long and 9^{Dm} 8^{m} wide?

3. If a piece of paper is 8^{Dm} wide, how long must it be to contain $2^{sq\,Dm}$ $40^{sq\,m}$?

4. How many centares are there in a sidewalk 1^{Km} 7^{Hm} 5^{Dm} long and 1^{m} 8^{dm} wide? How many ares?

5. How many tiles 2^{dm} square will be required to pave a corridor 8^{m} 6^{dm} long and 1^{m} 8^{dm} wide?

6. A walk 9^{dm} wide contains $1^{sq\,Dm}$ $26^{sq\,m}$. Find the length of the walk in meters.

7. How many ares are there in a rectangular field 6^{Dm} 2^{m} long and 4^{Dm} 3^{m} 6^{dm} wide?

8. A rectangular field 4^{Hm} 8^{Dm} wide contains 36^{Ha}. Find the length of the field in dekameters.

9. How many square meters are there in the walls and ceiling of a room 6^{m} 8^{dm} by 5^{m} 7^{dm} by 3^{m} 2^{dm}?

10. At $85 a hectare, what is the value of a piece of land 1^{Km} 2^{Hm} 8^{Dm} long and 7^{Hm} 5^{Dm} wide?

11. How many brick 2^{dm} by 8^{cm} will be required to pave a walk 8^{Hm} 6^{Dm} long and 1^{m} 8^{dm} wide?

12. At $1.45 a dekameter, what will it cost to build a fence on both sides of a road 8^{Hm} 4^{Dm} long?

13. A rectangular field 8^{Hm} 6^{Dm} long contains 55^{Ha} 90^{a}. Find the length of the field in dekameters.

VOLUME MEASUREMENTS

The units of volume measurements are cubes whose dimensions are the corresponding units of length. The names of the units are the same as those used in linear measure with the word *cubic* prefixed. A cubic meter is a cube each of whose edges is 1 meter. Since a cubic meter is 10 dm long, 10 dm wide, and 10 dm thick, it contains 1000 cu dm. Thus each unit of volume is 1 thousandth of the next higher and 1000 of the next lower. Hence, in reducing any number of cubic units to the next lower, we multiply by 1000; in reducing any number of units to the next higher, we divide by 1000.

$$5374582^{\text{cu cm}} = 5374.582^{\text{cu dm}} = 5.374582^{\text{cu m}}$$

In measuring wood, the cubic meter is called a *stere*.

1. How many cubic meters of air are there in a room 6^{m} long, 4^{m} 6^{dm} wide, and 3^{m} high?

2. How long must a stick of timber be that is 2.5^{dm} square at the ends to contain $.625^{\text{cu m}}$?

3. A pile of wood is 2^{m} wide and 1^{m} 2^{dm} 5^{cm} high. If it contains 24 steres, how long is it?

4. At $2 a stere, what is the value of a pile of wood 24^{m} long, 2^{m} 6^{dm} wide, and 1^{m} 8^{dm} high?

5. Find the volume in cubic decimeters of a block of marble 2^{m} 6^{dm} long, 1^{m} 2^{dm} wide, and 1^{m} thick?

MEASURES OF CAPACITY

The unit of capacity in both liquid and dry measure is the *liter* ($^{\text{l}}$), which is equal to a cubic decimeter.

Each unit is 1 tenth of the next higher and 10 of the next lower. Hence, in reducing any number of units of capacity to the next lower, we multiply by 10; in reducing any number of units to the next higher, we divide by 10.

$$6475^{\mathrm{dl}} = 647.5^{\mathrm{l}} = 64.75^{\mathrm{Dl}} = 6.475^{\mathrm{Hl}}$$

1. Express $65^{\mathrm{Hl}}\ 6^{\mathrm{l}}\ 54^{\mathrm{cl}}$ as liters. Express $42.8^{\mathrm{cu\,m}}$ as liters; as hektoliters.

2. How many hektoliters of wheat does a bin hold that is 5^{m} long, $3^{\mathrm{m}}\ 6^{\mathrm{dm}}$ wide, and $2^{\mathrm{m}}\ 6^{\mathrm{dm}}$ deep?

3. Find the capacity in liters of a cistern that is $4^{\mathrm{m}}\ 2^{\mathrm{dm}}$ long, 3^{m} wide, and $2^{\mathrm{m}}\ 8^{\mathrm{dm}}$ deep.

4. What must be the length of a tank that is 1^{m} wide and 1^{m} deep to contain 3800^{l}?

5. How many hektoliters of air are there in a room 7.2^{m} long, $5^{\mathrm{m}}\ 8^{\mathrm{dm}}$ wide, and 3.2^{m} high?

6. If a rectangular cistern 2^{m} long and 1.5^{m} wide holds 25^{Hl} of water, how deep is it?

7. Find the capacity in hektoliters of a cubical bin that is $4^{\mathrm{m}}\ 8^{\mathrm{dm}}$ deep.

8. A bin holds 36^{Hl} of wheat. If it is 4^{m} long and 3^{m} wide, how many decimeters deep is it?

WEIGHT

The primary unit of weight is the *gram*, which is the weight of a cubic centimeter of pure water.

Each unit is 1 tenth of the next higher and 10 of the next lower. Hence, in reducing any number of units of weight to the next lower, we multiply by 10; in reducing any number of units to the next higher, we divide by 10.

$$46325^{\mathrm{dg}} = 4632.5^{\mathrm{g}} = 463.25^{\mathrm{Dg}} = 46.325^{\mathrm{Hg}}$$

The unit of weight for heavy articles is the tonneau, which is the weight of a cubic meter of water, or 1000 kilograms.

The pupil should also remember that a cubic decimeter of water, or a liter, weighs a kilogram.

1. A cubical cistern 3.8^{m} deep is full of water. Find the weight of it in kilograms.

2. If mercury is 13.5 times as heavy as water, what is the weight of 219^{dl} of mercury?

3. How many decigrams of water will a pan hold that is 1^{m} long, 6^{dm} wide, and 2.5^{dm} deep?

4. A pan is 1^{m} long and 4^{dm} wide. How deep must it be to hold 90^{l} of water?

5. How long must a pile of wood be to contain 133 steres, if it is 3^{m} 8^{dm} wide and 2^{m} 5^{dm} high?

6. How long will it take a pipe to fill a cubical cistern 3^{m} 4^{dm} deep, if 2.6^{Dl} flow in per minute?

7. Find the capacity in hektoliters of a bin 8.2^{m} long, 3.5^{m} wide, and 2^{m} 8^{dm} deep.

8. How many brick 20^{cm} long and 10^{cm} wide will it take to pave a sidewalk 2.4^{m} wide and 1.5^{Km} long?

9. A block of wood contains $960000^{cu\,cm}$. If it is 8^{dm} wide and 6^{dm} thick, what is the length in meters?

10. If copper is 8.8 times as heavy as water, what is the weight in dekagrams of $6^{cu\,dm}$ of the metal?

11. The capacity of a tank is $60^{cu\,m}$. How long will it take a pipe to fill it at the rate of 3.8^{Dl} a minute?

12. If $1^{cu\,dm}$ of air weighs $.0018^{Kg}$, find the weight of the air in a room 5^{m} long, 4^{m} wide, and 3^{m} high.

13. How many liters of water will a vat hold that is 3^{m} long, 1^{m} 8^{dm} wide, and 9^{dm} 8^{cm} deep? How many hektoliters? Find the weight of the water in kilograms.

MENSURATION

It is of great importance that pupils should illustrate, with the aid of the teacher, everything defined in mensuration.

A **Line** is that which has length only. A **Straight Line** is a line that does not change its direction. A **Curved Line** is a line that changes its direction at every point.

Parallel Lines are lines that are equally distant from each other throughout their entire length.

An **Angle** is the opening between two lines that meet.

The **Vertex** of an angle is the point where the two sides of the angle meet.

A **Perpendicular Line** is a straight line that meets another line making the two angles equal.

A **Right Angle** is an angle formed by two lines that are perpendicular to each other. An **Acute Angle** is an angle that is smaller than a right angle. An **Obtuse Angle** is an angle that is larger than a right angle.

A **Surface** is that which has length and breadth, but no thickness. A **Plane Surface** is a surface that does not change its direction.

A **Plane Figure** is a portion of a plane surface bounded by straight or curved lines.

A **Polygon** is a plane figure bounded by three or more straight lines. The **Perimeter** is the sum of its sides.

A **Quadrilateral** is a plane figure bounded by four straight lines. A **Parallelogram** is a quadrilateral whose opposite sides are parallel. A **Trapezoid** is a quadrilateral which has two of its sides parallel. A **Trapezium** is a quadrilateral which has no two sides parallel.

When all the sides of a plane figure are equal, the figure is said to be *equilateral.*

A **Rectangle** is a parallelogram whose angles are right angles. A **Square** is an equilateral rectangle.

A **Rhomboid** is a parallelogram whose angles are not right angles. A **Rhombus** is an equilateral rhomboid.

The **Diagonal** of a plane figure is a straight line joining the vertices of two angles not adjacent.

The **Area** of any surface is the number of square units that it contains.

AREA OF RECTANGLES AND SQUARES

We have learned to find the area of rectangles and squares by *multiplying the length by the width.*

1. How many square yards are there in a piece of ground 264 feet long and 177 feet wide?

2. How many acres are there in a rectangular field 96 rods long and 55 rods wide?

3. How many acres are there in a rectangular field 28 ch. 75 l. long and 16 ch. 84 l. wide?

AREA AND ONE DIMENSION

Since the area, or the number of square units in any surface, is the product of the two dimensions, if the area and one dimension are given, the other dimension is found by *dividing the area by the given dimension.*

Note.—The area and given dimension must be expressed in corresponding units. If the area is expressed in square yards and the given dimension in feet, the area must be reduced to square feet; if the area is expressed in acres and the given dimension in rods, the area must be reduced to square rods; if the area is expressed in acres and the given dimension in chains, the area must be reduced to square chains.

1. A rectangular field 64 rods long contains 14$\frac{2}{5}$ acres. Find the width of the field.

2. A rectangular field 22 ch. 50 l. long contains 37.8 acres. Find the width of it.

3. A rectangular piece of ground 130 feet wide contains 2816$\frac{2}{3}$ square yards. Find the length.

In finding the area of a square, we multiply the side of the square by itself, or square it. The side of the square is therefore the square root of the area. Hence, if the area of a square is known, the side of it may be found by *finding the square root of the area.*

If the area is expressed in acres, it must be reduced to square rods or square chains before getting the square root.

1. The area of a square field is 10 acres. How long is each side?

2. What is the perimeter of a square field whose area is 4 acres?

3. What are the dimensions of a square ceiling whose surface is 27 square yards?

4. What is the perimeter of a square whose area is 27$\frac{9}{16}$ square feet?

5. What is the perimeter of a square garden whose area is 56$\frac{1}{4}$ square rods?

6. The area of a rectangle is 1440 square feet. What is the side of a square of equal area?

7. At $1.25 a rod, what will it cost to build a fence around a square field that contains 40 acres?

8. A man owns a square farm containing 45 acres 25 square rods. How many rods of fence will enclose it?

TRIANGLES

A **Triangle** is a plane figure bounded by three straight lines.

The **Base** of a triangle is the side on which it is supposed to stand.

The **Vertical Angle** of a triangle is the angle opposite the base.

The **Altitude** of a triangle is the perpendicular distance from the vertex of the vertical angle to the base, or to the base prolonged.

The **Dimensions** of a triangle are its base and altitude.

It is proved in geometry that any triangle is equal to half of a rectangle having the same base and altitude. Since the area of the rectangle is the product of the two dimensions, the area of the triangle is the product of one dimension and half the other.

Find the area of a triangle whose base is 50 feet and whose altitude is 18 feet.

$50 \times 9 = 450.$ $25 \times 18 = 450.$ Area 450 sq. ft.

1. Find the area in acres of a triangular field whose base is 64 rods and altitude 35 rods.

2. Find the area in acres of a triangular field whose base is 25 chains and altitude 18 chains.

3. A triangular piece of land contains $2\frac{1}{4}$ acres. If the altitude is 24 rods, what is the base?

4. The area of a triangular surface is 24 square yards, and the altitude is 18 feet. Find the base.

5. At $145 an acre, find the value of a triangle of land whose base is 32 rods and altitude 28 rods.

A **Right Triangle** is a triangle having one right angle.

The **Hypotenuse** of a right triangle is the side opposite the right angle.

The square of the hypotenuse is equal to the sum of the squares of the other sides. Hence,

To find the hypotenuse of a right triangle:

RULE. — *Square the two sides about the right angle, add the squares, and find the square root of the sum.*

When the hypotenuse and one side are known, to find the other side, *we square the hypotenuse and the given side, subtract the smaller square from the larger, and find the square root of the difference.*

1. The two sides about the right angle in a right triangle are 20 in. and 14 in. Find the hypotenuse.

2. The hypotenuse of a right triangle is 25 in., and one side is 14 in. Find the other side.

3. How long is the diagonal of a rectangle that is 18 rods long and 12 rods wide?

4. The longest side of a right triangle is 12 ft., and one other side is 8 ft. Find the third side.

5. How long is the longest straight line that can be drawn on a sheet of paper 20 in. long and 15 in. wide?

6. Two sides of a right triangle are each 30 inches. What is the length of the other side?

7. What distance will a man travel in going diagonally across a rectangular field 160 rd. by 60 rd.?

8. A ladder 60 feet long stands 12 feet from a building, and the upper end rests against the building 6 feet from the top. What is the height of the building?

To find the diagonal of a square, we would first square the two sides about the right angle. Each of these squares would be the area of the square, and their sum would be twice the area. Hence, to find the diagonal of a square when the area is known, we *multiply the area by* 2 *and find the square root of the product.*

1. How long is the diagonal of a square whose area is 225 square inches?

2. How long is the side of a square whose area is 156.25 square rods?

3. How long is the diagonal of a square whose area is 625 square rods?

4. The diagonal of a square is 14 inches. What is the dimension of the square?

5. The area of a square field is 10 acres. Find the distance from one corner to the opposite corner.

6. Find the length of the longest line that can be drawn on a rectangular surface 16 ft. by 3 yd.

7. How long is the side of a square whose area is equal to the area of a rectangle 49 ft. by 36 ft.?

8. The area of a square lawn is 144 square rods. Find the distance in feet from the center to each corner.

9. The diagonal of a rectangle is 14 ft., and one side is 12 ft. Find the other dimension of the rectangle.

10. A pole standing 40 ft. from a building was broken off at the ground, and in falling, it struck the building 50 ft. from the ground. Find the length of the pole.

11. If it costs $350 to enclose a rectangular field 80 rd. by 20 rd., how much will it cost to enclose a square field of the same area with the same kind of fence?

AREA OF TRIANGLE WHEN THREE SIDES ARE GIVEN

RULE. — *Find half the sum of the sides; from this subtract each side separately, find the product of the half sum and the remainders, and extract its square root.*

1. Find the area of a triangular surface whose sides are 16, 20, and 22 ft., respectively.

2. What is the area of an equilateral triangle each of whose sides is 14 feet?

3. Two sides of a right triangle are each 25 feet. How long is the other side?

4. What is the area of an equilateral triangle the sum of whose sides is 48 feet?

5. The perimeter of an equilateral triangle is 72 feet. Find the area of the triangle.

6. The two sides about the right angle in a right triangle are 40 ft. and 50 ft. Find the other side.

7. How many square yards are there in a triangle whose base is 72 ft. and whose altitude is 54 ft.?

8. Find the width and area of a rectangular field whose length is 20 rods and whose diagonal is 25 rods.

9. Find the length of the longest straight line that can be drawn on a square whose area is 185 square inches.

10. How many square rods are there in a triangular field whose sides are 16, 18, and 20 rods, respectively?

11. At $1.32 a square yard, it cost $19.80 to pave a triangular court whose base is 18 ft. Find the altitude.

12. A ladder 30 feet long stands close against the side of a building. How far must the foot of the ladder be moved away that the top may be lowered 5 feet?

AREA OF A PARALLELOGRAM

The **Altitude** of a parallelogram is the perpendicular distance from the base to the opposite side.

The area of any parallelogram is equal to the product of the base and altitude.

1. Find the area of a parallelogram whose base is 75 yards and whose altitude is 175 feet.

2. The base of a rhomboid is 85 feet, and the altitude is 48 feet. What is the area?

3. What is the area of a rhombus whose perimeter is 64 rods and whose altitude is 15 rods?

4. A rhomboid of land whose base is 24.8 chains contains 43.4 acres. What is the altitude?

AREA OF A TRAPEZOID

The area of a trapezoid is equal to half the sum of the parallel sides multiplied by the altitude.

1. Find the area of a trapezoid whose parallel sides are 142 ft. and 116 ft. and whose altitude is 48 ft.

2. The parallel sides of a trapezoid are 14 ft. and 19 ft., and the altitude is 8 ft. What is the area?

3. Find the area of a trapezoid whose parallel sides are 84 rods and 66 rods and whose altitude is 38 rods.

4. At $2.50 a square foot, what is the value of a piece of land in the form of a trapezoid, if the parallel sides are 16 yards and 34 feet and the altitude 18 feet?

5. Two of the boundary lines of a field run east and west and are 70 rods and 58 rods in length. The distance between these sides is 42 rods. Find the area in acres.

AREA OF A TRAPEZIUM

Any trapezium is divided into two triangles by the diagonal, and the diagonal may be regarded as the base of each. The altitude of each triangle is the perpendicular from the vertex of the opposite angle to the diagonal. Hence, to find the area of a trapezium, we have the rule:

Multiply the diagonal by half the sum of the perpendiculars drawn to it from the vertices of the opposite angles.

1. The diagonal of a trapezium is 28 inches and the perpendiculars from the vertices of the opposite angles 18 inches and 16 inches. What is the area?

2. The diagonal of a quadrilateral is 18 yards, and the perpendiculars from the vertices of the opposite angles are 18 feet and 14 feet. What is the area?

3. One side of a quadrilateral is 34 feet, the side opposite and parallel to it 28 feet, and the altitude 19 feet. How many square yards does it contain?

4. Find the area of a quadrilateral whose diagonal is 18 feet, the perpendiculars from the opposite angles to the diagonal being 8 feet and 7 feet.

5. At $125 an acre, what is the value of a piece of land in the form of a trapezoid whose parallel sides are 60 rods and 40 rods and whose altitude is 32 rods?

6. Find the area of a quadrilateral whose diagonal is 4 rods, the perpendiculars from the vertices of the opposite angles to the diagonal being 6 yards and 22 feet.

7. Two of the fences enclosing a field run north and south and are 32 rods and 28 rods in length. The distance between them is 16 rods. Find the area in acres.

CIRCLES

A **Circle** is a plane figure bounded by a curved line every point of which is equally distant from the center.

The **Circumference** of a circle is the curved line that bounds it.

The **Diameter** of a circle is a straight line drawn through the center terminating in the circumference.

The **Radius** of a circle is a straight line drawn from the center to the circumference.

A **Circumscribed Square** is a square formed about a circle, the sides of which touch the circumference.

An **Inscribed Square** is a square formed in a circle and having the vertices of its angles in the circumference.

It is proved in geometry that the circumference of a circle is 3.1416 times the diameter.

1. What is the circumference of a circle whose diameter is 18 feet? Of one whose radius is 34 feet?

2. What is the diameter of a circle whose circumference is 26 rods? What is the radius?

3. What is the circumference of a circle whose radius is 14 feet? Of one whose diameter is 28 yards?

4. What is the diameter of a circle whose circumference is 16 rods? What is the radius?

5. What is the length of a tire on a wheel that is 4 feet 9 inches in diameter?

6. What is the circumference of the largest circle that can be drawn on a board 14 inches square?

7. Find the circumference of the largest circle that can be drawn on a sheet of paper 14 in. long and 9 in. wide.

AREA OF A CIRCLE

A circle may be regarded as composed of a large number of triangles, the sum of whose bases is the circumference of the circle, and whose altitude is the radius.

The area of a circle is equal to the product of the circumference and half the radius.

Half the radius is a fourth of the diameter.

The area of a circle is equal to the square of the diameter multiplied by .7854.

When only the diameter of a circle is known, it is more convenient to find the area by this principle.

1. Find the area of a circle whose diameter is 4 feet.

2. What is the area of a circle whose radius is $3\frac{1}{2}$ feet?

3. The circumference of a circle is 2 rods. How many square feet does it contain?

4. Find the area in square rods of a circle whose circumference is one mile.

5. The circumference of a circle is 125 yards. What is the area in square feet?

6. What is the area of a semicircle, if the diameter of the circle is 48 feet?

7. How many square feet are there in a circle whose radius is 15 yards?

8. The circumference of a circle is 12 rods 7 feet. How many square feet does it contain?

9. What is the circumference of the largest circle that can be drawn on a rectangle 48 inches wide?

10. The perimeter of a square and the circumference of a circle are each 260 feet. Find the difference in area.

Since the area of a circle is the product of the square of the diameter and .7854, the diameter is found by dividing the area by .7854 and finding the square root of the quotient.

1. What is the diameter of a circle whose area is 39.27 square rods?

2. If the area of a circle is 78.54 square feet, what is the radius of the circle?

3. Find the circumference in rods of a circle whose area is half an acre.

The diagonal of an inscribed square is the diameter of the circle. It is also the hypotenuse of a right triangle the other two sides of which are sides of the square. The square of the diameter is therefore twice the square of the side of the square. Hence, to find the side of the inscribed square, *square the diameter, divide the square by* 2, *and find the square root of the quotient.*

The side of a circumscribed square is equal to the diameter.

1. Find the side of a square inscribed in a circle whose diameter is 14 inches.

2. How long is the side of a square inscribed in a circle 18 feet in diameter?

3. How many square yards are there in a semicircle whose diameter is 29 feet?

4. The circumference of a circle is 17 yards. What is the area in square feet?

5. Find the side of the largest square that can be cut from a circular piece of paper 22 inches in diameter.

6. What is the difference in the areas of two squares, one circumscribing a circle 35 inches in diameter, and the other inscribed in the same circle?

PRISMS AND CYLINDERS

A **Solid** is that which has length, breadth, and thickness. A **Rectangular Solid** is a solid bounded by six rectangular faces. The **Dimensions** of a rectangular solid are its length, breadth, and thickness.

A **Prism** is a solid whose ends are similar, equal, and parallel figures and whose sides are parallelograms.

A **Square Prism** is a prism whose ends are squares. A **Triangular Prism** is a prism whose ends are triangles.

A **Cylinder** is a solid bounded by a uniformly curved surface, the ends being equal and parallel circles.

The **Altitude** of a prism or cylinder is the perpendicular distance between its ends.

The **Lateral Surface** of a prism or cylinder is all its surface except that of the ends.

The lateral surface of a prism or cylinder is a rectangle whose length is the perimeter of the base and whose width is the altitude of the prism or cylinder.

1. Find the lateral surface of a cylinder whose circumference is 4½ feet and whose altitude is 15 feet.

2. What is the lateral surface of a cylindrical granite column 2 feet in diameter and 18 feet long?

3. Find the entire surface of a square prism the perimeter of whose base is 32 in. and whose altitude is 17 in.

4. Find the entire surface of a cylinder whose diameter is 3 feet and whose altitude is 14 feet.

5. What is the entire surface in square inches of an equilateral triangular prism the perimeter of whose base is 27 inches and whose altitude is 5 feet?

The volume of a prism or cylinder is the product of the area of the base and the altitude.

1. How many cubic inches are there in a cylinder whose diameter is 8 inches and whose altitude is 3 feet?

2. How many cubic feet are there in a stick of timber 24 feet long and 14 inches square?

3. Find the capacity in gallons of a cylindrical cistern 8 feet in diameter and 6 feet deep.

4. Find the volume of a square prism the perimeter of whose base is 3 feet and whose altitude is 16 feet.

5. Find the entire surface of a cylinder whose circumference is 6.2832 feet and whose altitude is 18 feet.

6. What is the volume in cubic inches of an equilateral triangular prism the perimeter of whose base is 3 feet and whose altitude is 8 feet?

7. What is the lateral surface of a cylinder formed by the revolution of a rectangle about one of its sides, if the rectangle is 9 feet long and 4 feet wide?

PYRAMIDS AND CONES

A **Pyramid** is a solid whose base is a polygon and whose faces are triangles.

A **Square Pyramid** is a pyramid whose base is a square. A **Triangular Pyramid** is one whose base is a triangle.

A **Cone** is a solid whose base is a circle and whose surface tapers uniformly to a point called the vertex.

The **Altitude** of a pyramid or cone is the perpendicular distance from the vertex to the plane of the base.

The **Slant Height** of a pyramid or cone is the shortest distance from the vertex to the perimeter of the base.

The lateral surface of a pyramid or cone is the product of the perimeter of the base and half the slant height.

The volume of a pyramid or cone is the product of the area of the base and one-third the altitude.

1. Find the lateral surface of a cone whose base is 2 ft. in circumference and whose slant height is 56 in.

2. Find the entire surface of a square pyramid the perimeter of whose base is 20 ft. and slant height 12 ft.

3. Find the lateral surface of a pyramid whose base is 8 in. square and whose slant height is 28 in.

4. Find the entire surface of a cone the diameter of whose base is 3 ft. and whose slant height is 14 ft.

5. Find the volume of a square pyramid the perimeter of whose base is 8 ft. and whose altitude is 18 ft.

6. Find the volume of a cone the circumference of whose base is 27 in. and whose altitude is 4 ft.

7. Find the volume of a pyramid whose base is 5 feet square and whose altitude is 21 feet.

8. What is the lateral surface of a pyramid whose base is 6 feet square and whose altitude is 15 feet?

9. Find the entire surface of a cone the radius of whose base is 2 ft. 6 in. and whose slant height is 24 ft.

10. What is the volume of an equilateral triangular prism 12 feet high, the sides of whose base are each 2 feet?

11. How much will it cost, at 35 cents a square yard, to paint a church spire whose base is a hexagon 8 feet on each side and whose slant height is 75 feet?

12. How many square yards are there in the entire surface of a rectangular pyramid whose base is 5 feet long and 3 feet wide and whose slant height is 18 feet?

SPHERES

A **Sphere** is a solid bounded by a curved surface every point of which is equally distant from the center.

The **Diameter** of a sphere is a straight line passing through the center terminating in the surface.

The **Circumference** of a sphere is the greatest distance around it.

The surface of a sphere is the product of the square of the diameter and 3.1416.

The volume of a sphere is the product of the surface and one-sixth the diameter.

1. What is the area of a sphere whose diameter is 16 inches? Of one whose radius is 14 inches?

2. What is the area in square inches of a sphere whose diameter is 2 feet?

3. What is the volume of a sphere whose diameter is 12 inches? Of one whose radius is 9 inches?

4. Find the capacity in gallons of a hollow sphere the inside diameter of which is one yard.

5. What is the area of a sphere whose circumference is one yard? Of one whose radius is 18 inches?

6. The surface of a sphere is the same as that of a cube 2 feet on each edge. Find the volume of each.

7. What is the perimeter of a square circumscribed about a circle whose area is one acre?

8. Find the length of the longest straight line that can be drawn on a square whose area is 156 sq. in.

9. Find the side of the largest square that can be cut from a circular sheet of tin 27 inches in diameter.

1. What is the volume of a cube whose entire surface contains 1176 square inches?

2. How long is the side of a square inscribed in a circle whose area is 75 square feet?

3. How many square feet are there in an equilateral triangle whose perimeter is 72 feet?

4. What is the circumference of the largest circle that can be drawn on a paper 45 inches square?

5. A pole 80 ft. high stands 200 ft. from a post 10 ft. high. Find the distance between their tops.

6. Find the volume of the largest cone that can be cut from a prism 18 inches square and 4 feet long.

7. What is the volume of the largest pyramid that can be cut from a prism 4 ft. square and 8 ft. long?

8. How long a cord will reach from a lower corner to the opposite upper corner of a room $14' \times 12' \times 9'$?

9. The volume of a cubical stone is 64,000 cubic inches. How many square inches are there in its surface?

10. The longest straight line that can be drawn on a given square is 14 feet. Find the area of the square.

11. The parallel sides of a trapezoid are 22 rods and 86 rods and the altitude 18 rods. Find the area in acres.

12. The perimeter of a square and the circumference of a circle are each 60 feet. Find the difference in area.

13. Two of the bounding lines of a field run north and south and are 80 rods and 68 rods in length. The distance between them is 58 rods. Find the area in acres.

14. Find the area in square feet of a quadrilateral whose diagonal is 6 rods, the perpendiculars from the opposite angles to the diagonal being 14 yards and 38 feet.

ALGEBRA

Letters are used to represent numbers, but they do not, like figures, always represent the same number. The letter x, for example, may represent 4 in one problem, 12 in another, 25 in another, and so on. When a figure is written with a letter, without any sign between them, the figure indicates how many times the number represented by the letter is taken; and when one letter is written with another, either letter indicates how many times the number represented by the other letter is taken. For example, whatever number a letter represents,

$4a$ means 4 times a — ax means a times x

$8x$ means 8 times x — xy means x times y

The expression $4x$ rods means 4 times the number of rods represented by x.

1. What do you mean by the expression, $7x$ yards? $9x$ dollars? $x+8$ days?

2. If x represents the price of a yard of silk, what does $8x$ represent? $5x$? ax?

3. Indicate the sum of a and b, diminished by c. The sum of x and $3x$, diminished by y.

4. Indicate in two ways the product of 5 and x. Of b and y. Of 5, a, and b. Of x, y, and 3.

5. What do you mean by the expression, $5x$ miles? $8x$ bushels? $3x+7$ pounds? $8x-9$ inches?

6. Indicate the difference between a and b, when a is greater than b. When b is greater than a.

7. A man paid x dollars for a harness and $4x$ dollars for a horse. What will represent the cost of both?

1. What do you mean by the expression, $9x$ quarts? $12x$ cents? $8x$ square rods?

2. If a yard of ribbon is worth x cents, how much is a foot worth? Two feet?

3. A lady paid a dollars for silk at x dollars a yard. How many yards did she buy?

4. A man is 3 times as old as his son. If the son is x years old, how old is the father?

5. A man sold a horse for x dollars, thereby gaining y dollars. Find the cost of the horse.

6. What do you mean by the expression, $6x$ yards? $45x$ dollars? $7x$ square yards?

7. If a rectangle is $3x$ rods long and $2x$ rods wide, what will represent the perimeter?

8. A boy has $4x$ dimes. How many oranges can he buy at 30 cents a dozen?

9. How many square yards are there in a ceiling x feet long and y feet wide?

10. A man had x dollars and paid two debts of a dollars and b dollars. How much had he left?

11. A grocer bought x barrels of flour for a dollars. What was the price of the flour per barrel?

In the equation, $3x = 18$, since $3x$ means 3 times x, $3x$ and 18 are equal only when x represents 6.

In the equation, $3x + 2x = 35$, since $3x + 2x$ means 5 times x, $3x + 2x$ and 35 are equal only when x represents 7.

In the equation, $5x - x = 36$, since $5x - x$ means 4 times x, $5x - x$ and 36 are equal only when x represents 9.

In such equations, x represents *some particular number*. The number represented by x in an equation is called the *unknown number*.

1. If $7x = 28$, what does x represent? What does x represent in the equation, $3x + x = 24$?

2. If $2x + 3x = 27$, what does x represent? What does x represent in the equation, $2x - x = 17$?

3. If one part of x is 7, what is the other part? If one part of x is a, what is the other part?

4. If $4x + 3x + x = 32$, what does x represent? What does x represent in the equation, $7x - 3x - x = 36$?

5. A boy bought x apples at a cents apiece and sold them at b cents. Find the gain, if a is greater than b.

6. A man worked 8 days of x hours each at y cents an hour. He spent a dollars. How much did he have left?

SOLUTION OF PROBLEMS BY ALGEBRA

The sum of two numbers is 595, and the larger number is 6 times the smaller. Find the two numbers.

Let $x =$ the smaller number;
then $6x =$ the larger number.

Since the *sum* of the numbers is 595, $6x + x = 595$, and

$$7x = 595$$
$$x = 85, \text{ the smaller number;}$$
$$6x = 510, \text{ the larger number.}$$

1. A has three times as many sheep as B, and both have 276. How many has A?

2. A is six times as old as B, and the difference between their ages is 65 years. Find the age of B.

3. There are 795 pupils in a school, and there are twice as many girls as boys. How many girls are there?

4. The sum of two numbers is 126, and the larger number is 8 times the smaller. Find the larger number.

1. The sum of two numbers is 112, and their difference is 36. Find the larger number.

2. Twice a number, increased by 68, is equal to 3 times the number, diminished by 57. Find the number.

3. A and B own a farm worth $13,100. A has 3 times as much invested as B. How much has B invested?

4. A farm cost 4 times as much as a house. If the farm cost $8400 more than the house, what did both cost?

5. A has twice as many sheep as C, and B has four times as many as C. If all have 595, how many has each?

6. My house cost 3 times as much as the lot. One cost $5000 less than the other. What did the house cost?

7. A and B are 81 miles apart. They travel toward each other until they meet, A traveling twice as many miles as B. How many miles did B travel?

8. A has twice as many acres of land as B, and B has three times as many acres as C. If all of them have 2600 acres, how many acres have B and C together?

9. A, B, and C own 750 acres of land. B owns three times as many acres as A, and C owns half as many acres as A and B together. How many acres has C?

10. A man's salary was doubled each year for 3 years. If he received five thousand two hundred fifty dollars in the 3 years, what was his salary the second year?

11. A, B, C, and D have 280 sheep. B has twenty more than A, C has twenty more than B, and D has twenty more than C. How many have A and B?

12. The sum of the ages of father, mother, and son is 105 years. The mother is twice as old as the son, and five years younger than the father. Find the father's age.

The **Algebraic Notation** is the method of expressing numbers by letters and figures. Since letters are used, the algebraic notation is called a literal notation.

An **Algebraic Expression** is the representation of any number in the algebraic notation.

$2a + 3b - 4c$ $4xy$ $2a + 5b$ $8x - 6y + 4z$

A **Term** is an algebraic expression whose parts are not separated by the sign of addition or subtraction.

$4a \times 2b$ $5abc$ $4xyz$ $5x \div 2y$

A **Coefficient** is a factor of a term. The numerical factor of a term is generally thought of as the coefficient, but any factor, or the product of any factors, may be considered the coefficient of the rest of the term. When no numerical coefficient is expressed, 1 is understood.

In the term $4xy$, 4 is the coefficient of xy; $4x$ is the coefficient of y; $4y$ is the coefficient of x.

In the term bcx, 1 is the coefficient of bcx; b is the coefficient of cx; bc is the coefficient of x.

An **Exponent** is a symbol of number written at the right and a little above another symbol of number. When no exponent is expressed, 1 is always understood.

$$a^4b^3c^2 = a \times a \times a \times a \times b \times b \times b \times c \times c$$

Similar Terms are terms having the same letters and the corresponding letters affected by the same exponent.

$5x^2y$, x^2y, $8x^2y$ $3a^3b$, $7a^3b$ $5xy^2$, xy^2, $8xy^2$

The **Numerical Value** of an algebraic expression is the number obtained by assigning a numerical value to each letter and performing the indicated operations.

Find the numerical value of the following expressions when $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 0$, $m = \frac{1}{2}$, and $n = \frac{1}{3}$:

1. $9c - 9n + 8d + 7c$.
2. $5c + 4b - 6n + 6m$.
3. $5a + 4b + 3c + 8d$.
4. $8d - 7a + 8m + 6c$.
5. $7a + 5d - 3n + 6e$.
6. $9d - 8e + 4m - 3a$.
7. $8b + 6c - 2d + 9c^2$.
8. $4c + 6e + 8b^3 + 9a^3$.
9. $7d + 6m + 8c - 9n^2$.
10. $8m + 4e + 5d - 9a^2$.

POSITIVE AND NEGATIVE NUMBERS

Some numbers in their relation to each other are opposite in character, for when they are combined the numerically smaller one cancels part of the larger. Gains and losses, debits and credits, assets and liabilities, and distances in opposite directions are examples.

To describe the opposite quality of numbers, the terms *positive* and *negative* are used. The symbols used to denote the quality of numbers are those that indicate addition and subtraction, the plus sign denoting positive numbers, and the minus sign negative numbers.

The plus sign is usually omitted when it denotes quality. The minus sign is never omitted before a negative number.

ADDITION

Addition is the process of uniting two or more quantities into the simplest expression called their sum.

Since $6a$ is 6 times a, and $4a$ is 4 times a, the sum of $6a$ and $4a$ is 6 times a plus 4 times a, or 10 times a, or $10a$; for 6 times any quantity plus 4 times the same quantity is 10 times that quantity. If more than two positive terms are to be added, the reasoning is the same.

Since $-7a$ is 7 times $-a$ and $-5a$ is 5 times $-a$, the sum of $-7a$ and $-5a$ is 7 times $-a$ plus 5 times $-a$, or 12 times $-a$, or $-12a$; for 7 times any quantity plus 5 times the same quantity is 12 times that quantity. This reasoning applies to several terms.

$$\begin{array}{r} 4a \\ -2a \\ 7a \\ -6a \\ \hline 3a \end{array}$$

The sum of the positive terms is $11a$, and the sum of the negative terms is $-8a$. When these terms are combined, the $-8a$ cancels $8a$ of the $11a$, and the sum is $3a$.

$$\begin{array}{r} 5a \\ -8a \\ 3a \\ -9a \\ \hline -9a \end{array}$$

The sum of the positive terms is $8a$, and the sum of the negative terms is $-17a$. When these terms are combined, the $8a$ cancels $-8a$ of the $-17a$, and the sum is $-9a$.

$$\begin{array}{r} 2ab \\ ab \\ 4ab \\ \hline 7ab \end{array} \qquad \begin{array}{r} -3a^2b \\ -5a^2b \\ -\ \ a^2b \\ \hline -9a^2b \end{array} \qquad \begin{array}{r} 5ax \\ -3ax \\ ax \\ \hline 3ax \end{array} \qquad \begin{array}{r} -7ax^2 \\ -5ax^2 \\ 9ax^2 \\ \hline -3ax^2 \end{array} \qquad \begin{array}{r} -\ \ xy \\ 6xy \\ -\ \ xy \\ \hline 4xy \end{array}$$

RULE. — *If the signs are alike, add the coefficients, to their sum annex the literal part, and give the result the common sign.*

If the signs are unlike, add the positive terms and the negative terms separately, subtract the less sum from the greater, and give the result the sign of the greater.

1. $\begin{array}{r} 3ac \\ ac \\ 5ac \\ \hline \end{array}$ **2.** $\begin{array}{r} -5x^2 \\ -6x^2 \\ -\ \ x^2 \\ \hline \end{array}$ **3.** $\begin{array}{r} -8ab^2 \\ 6ab^2 \\ -5ab^2 \\ \hline \end{array}$ **4.** $\begin{array}{r} 7abc \\ -8abc \\ 9abc \\ \hline \end{array}$

5. $\begin{array}{r} xy \\ 9xy \\ xy \\ \hline \end{array}$ **6.** $\begin{array}{r} 7a^2 \\ -4a^2 \\ 8a^2 \\ \hline \end{array}$ **7.** $\begin{array}{r} -6x^2y \\ 5x^2y \\ -\ \ x^2y \\ \hline \end{array}$ **8.** $\begin{array}{r} 4acx \\ -\ \ acx \\ 3acx \\ \hline \end{array}$

9. $\begin{array}{r} 9ax \\ 4ax \\ 6ax \\ \hline \end{array}$ **10.** $\begin{array}{r} -7y^3 \\ y^3 \\ -2y^3 \\ \hline \end{array}$ **11.** $\begin{array}{r} a^4b \\ -5a^4b \\ 9a^4b \\ \hline \end{array}$ **12.** $\begin{array}{r} -4abx \\ abx \\ -6abx \\ \hline \end{array}$

1. $\begin{array}{r} 4\,a^2x \\ a^2x \\ 7\,a^2x \\ \hline \end{array}$

2. $\begin{array}{r} -5\,ax^2 \\ -2\,ax^2 \\ -\;\;ax^2 \\ \hline \end{array}$

3. $\begin{array}{r} 2\,a^2b^2 \\ -7\,a^2b^2 \\ 6\,a^2b^2 \\ \hline \end{array}$

4. $\begin{array}{r} -5\,abc \\ 8\,abc \\ -4\,abc \\ \hline \end{array}$

5. $\begin{array}{r} ac^2 \\ 8\,ac^2 \\ ac^2 \\ \hline \end{array}$

6. $\begin{array}{r} -6\,x^2y \\ 4\,x^2y \\ -8\,x^2y \\ \hline \end{array}$

7. $\begin{array}{r} -7\,x^2y^2 \\ 9\,x^2y^2 \\ -3\,x^2y^2 \\ \hline \end{array}$

8. $\begin{array}{r} -4\,axy \\ 7\,axy \\ -2\,axy \\ \hline \end{array}$

A **Polynomial** is an algebraic expression of two or more terms.

To add polynomials, write them so that similar terms shall be in a column and add each column separately.

$$\begin{array}{rrrrrrr} 5\,ab & +\,3\,ac & -\,2\,bc & +\,3\,bd & +\,5\,xy & -\,7\,z^2 & \\ 2\,ab & -\;\;ac & & -\,5\,bd & +\,2\,xy & & \\ & 5\,ac & -\;\;bc & & & +\,7\,z^2 & \\ ab & & +\,3\,bc & & -\,4\,xy & & +\,6 \\ \hline 8\,ab & +\,7\,ac & & -\,2\,bd & +\,3\,xy & & +\,6 \end{array}$$

1. Add $2x - 4y + 3z$, $-5z + y - 6x$, $3y + z + 5x$, $-8y + 4x - 4z$, and $7y + 3z - 4x$.

2. Add $5b + 3c - 6d$, $c - 2b + 3d$, $-4d - 5c + 3b$, $6d - 4c - 7b$, and $2c + d + 4b$.

3. Add $4ab - 2ac + 4bc$, $4ac - 2bc$, $-5ac - 2ab + 6bc$, $6ac - 9bc - 3ab$, and $ab - 2ac$.

4. Add $2a^2b + 3ab^2 - 4a^2b^2$, $2a^2b^2 - 5ab^2$, $4ab^2 - 5a^2b^2 - 7a^2b$, $9a^2b^2 + 4a^2b + 5ab^2$, and $2a^2b - 8ab^2$.

5. Add $7xy^2 - 4x^2y + 5x^2y^2$, $3x^2y^2 - 3xy^2$, $3x^2y - 7x^2y^2 + 5xy^2$, $4x^2y^2 - 7x^2y - 7xy^2$, and $9x^2y - 6x^2y^2 - xy^2$.

6. Add $4a^3 - 8a^2 + 3a - 5$, $3a^2 + 9 - 7a - a^3$, $-a^2 + 7a^3 - 7$, $6a^2 - 3a - 8a^3 + 6$, and $a - 3 + 3a^3$.

1. If you have x quarters and y dimes, how many cents have you?

2. If a man lives b years, he will be x years old. How old was he c years ago?

3. If you have x half-dollars and y quarters, how many dollars have you?

4. Represent the number in which there are x hundreds, y tens, and z units.

5. If x yards of silk cost a dollars, what will 24 yards cost at the same price?

6. How many square yards are there in an oblong x feet long and y feet wide?

7. How many acres are there in a rectangular piece of land x rods long and y rods wide?

8. What will represent the sum of 5 consecutive odd numbers of which x is the middle one?

9. The sum of the ages of three men is $4x$ years. What was the sum of their ages 6 years ago?

10. At a cents a rod, how many dollars will it cost to enclose a rectangular field x rd. by y rd.?

11. A boy bought x oranges at a cents apiece and sold them at b cents. If he lost, what was his loss?

12. A has x cows, B has 5 more than A, and C has as many as A and B together. How many have all?

13. If a field is x rods square, how many rods of fence will enclose it and divide it into 4 equal squares?

14. Find the value of x bushels of apples at a cents a peck and y bushels of pears at b cents a peck.

15. A man bought x sheep at a dollars a head and had b dollars left. How much money did he have at first?

TRANSPOSITION

To *transpose* a term in an equation is to take it out of one member of the equation and write in the other.

If the same number, or equal numbers, be added to equal numbers, the sums will be equal.

If in the equation, $7x - 4 = 8 + 5x$, we add 4 and $-5x$ to both members, we have $7x - 5x = 8 + 4$.

We observe that the last equation could have been obtained directly from the first by transferring -4 and $5x$ to the other member and changing their signs.

ORAL EXERCISES

1. $4x - 4 = 2x + 8$. **2.** $5x + 3 = 2x + 9$.

3. If x represents an odd number, what will represent the next larger odd number?

4. $9x - 8 = 4x + 7$. **5.** $7x + 4 = 3x + 8$.

6. If y represents an even number, what will represent the next smaller even number?

7. $6x - 3 = 5x + 4$. **8.** $8x + 1 = 6x + 9$.

9. If a man lives 8 years, he will be x years old. How old was he 8 years ago?

10. $1 + 2x = 9 - 2x$. **11.** $5 - 4x = 7 - 6x$.

12. If a man can perform a piece of work in x days, what part of it can he do in one day?

13. $3x - 7 = 8 - 2x$. **14.** $1 - 7x = 9 - 9x$.

15. What will represent the sum of three consecutive numbers of which x is the smallest?

16. What will represent the sum of four consecutive even numbers of which x is the largest?

1. Divide the number 148 into two such parts that the greater shall exceed the less by 34.

2. The sum of two numbers is 259, and their difference is 5 times the smaller. Find the larger number.

3. The length of a rectangle is 3 times the width, and the perimeter is 144 feet. Find the dimensions.

4. The length of a rectangle is 3 times its width, and the perimeter is 360 feet. Find the length.

5. A is 3 times as old as B and 8 years older than C. The sum of their ages is 90 years. Find C's age.

6. The sum of two numbers is 224, and their difference is 6 times the smaller. Find the larger number.

7. My house and lot cost $17,500, the house costing 4 times as much as the lot. Find the cost of the house.

8. A horse and carriage cost $365, the horse costing $85 more than the carriage. Find the cost of the horse.

9. A man's capital doubled for three successive years, when it amounted to $ 16,800. How much had he at first?

10. The sum of two numbers is 240. If the smaller number were doubled, it would still be 6 less than the larger number. Find the larger number.

11. A father and son earn $ 107 a month. If the son's wages were doubled, he would receive only $ 11 less than his father. How much does the son receive?

12. Three men raised 1925 bushels of corn. A had three times as many bushels as B, and 175 bushels more than C. How many bushels did A and B together raise?

13. In a company of 98 persons, it was found that there were twice as many women as men, and twice as many children as women. How many children were there?

SUBTRACTION

Subtraction is the process of finding the difference between two quantities. The **Minuend** is the quantity from which another quantity is to be subtracted. The **Subtrahend** is the quantity to be subtracted.

PRINCIPLE. — *The minuend is equal to the sum of the subtrahend and the difference.*

$$\begin{array}{rrrrrrrr} 9a & 3a & -9a & -3a & -9a & -3a & 9a & 3a \\ 3a & 9a & -3a & -9a & 3a & 9a & -3a & -9a \\ \hline 6a & -6a & -6a & 6a & -12a & -12a & 12a & 12a \end{array}$$

The above examples represent all possible cases in subtraction with reference to the signs and relative numerical value of minuend and subtrahend. The signs may be alike or unlike, and the minuend may be numerically greater or less than the subtrahend.

Since algebra deals with positive and negative quantities, and since the subtrahend may be either numerically less or *greater* than the minuend, it is evident that the method of subtraction pursued in arithmetic cannot in every case give the correct result.

It is easily determined by inspection of the above examples that $6a$, $-6a$, $-6a$, $6a$, $-12a$, $-12a$, $12a$, and $12a$ are the quantities which added to the several subtrahends will produce the minuends.

An examination of the work shows that each remainder could have been obtained by changing the sign of the subtrahend and adding.

Since the correct remainder, *in every example*, may be obtained in this manner, this method of subtraction is adopted.

RULE. — *Conceive the sign of the subtrahend to be changed from + to −, or from − to +, and add.*

$$\begin{array}{lrlrlrlr} \textbf{1.} & 8xy & \textbf{2.} & 4ac & \textbf{3.} & -5ax & \textbf{4.} & -4bc \\ & \underline{2xy} & & \underline{-3ac} & & \underline{6ax} & & \underline{-9bc} \\ \\ \textbf{5.} & 3ay & \textbf{6.} & -8cx & \textbf{7.} & -3ab & \textbf{8.} & 5xy \\ & \underline{7ay} & & \underline{-2cx} & & \underline{-9ab} & & \underline{-2xy} \end{array}$$

1. $\begin{array}{r} 9a^2 \\ \underline{3a^2} \end{array}$	**2.** $\begin{array}{r} -4ax^2 \\ \underline{-9ax^2} \end{array}$	**3.** $\begin{array}{r} -8a^2x \\ \underline{-2a^2x} \end{array}$	**4.** $\begin{array}{r} 4x^2y \\ \underline{-7x^2y} \end{array}$
5. $\begin{array}{r} 4x^2 \\ \underline{8x^2} \end{array}$	**6.** $\begin{array}{r} -3a^2b \\ \underline{6a^2b} \end{array}$	**7.** $\begin{array}{r} -2xy^2 \\ \underline{-7xy^2} \end{array}$	**8.** $\begin{array}{r} -9a^2c \\ \underline{-4a^2c} \end{array}$
9. $\begin{array}{r} 5b^2 \\ \underline{6b^2} \end{array}$	**10.** $\begin{array}{r} -4ax^3 \\ \underline{-5ax^3} \end{array}$	**11.** $\begin{array}{r} -8x^4y \\ \underline{-3x^4y} \end{array}$	**12.** $\begin{array}{r} 7a^3c \\ \underline{-3a^3c} \end{array}$

1. Subtract $4ax-2xy+7ab$ from $4ax-5ac-3xy+8ab$.

2. From $4ay-7xy-3by+5xz$ subtract $6xy+6xz-3by$.

3. Subtract $3a^2x-8-2a^3y$ from $3a^2x+3ax-2a^3y-4az$.

4. From $8x^2y-8xy^2+3xz$ subtract $7x^2y-2xz+7-9xy^2$.

5. Subtract $8x^2y-6xy-36x^3$ from $25x^3+9x^2y+7xy-8$.

6. From $7ab-5ab^2+24b^3$ subtract $7ab-6ab^2-7-37b^3$.

7. Subtract $7bc^2-abd+9b^2c$ from $4b^2c+8bc^2-abd-9$.

8. From $8a^2x-8a^2y+187$ subtract $z+7a^2y+243+7a^2x$.

9. Subtract $2xyz-93y^2-8+29x^3$ from $48x^3-7y^2+xyz$.

In such examples as the following write all the given polynomials, similar terms in a column, writing those to be subtracted with their signs changed, and then add.

1. From the sum of $2a+b-c+d$ and $3d+4a-3b$ subtract $2b+4+4d+7a$.

2. Subtract $3z-2u+2x$ from the sum of $3x-4y+3z-u$ and $8+5y-x-z$.

3. From $b-c+d-4e$ subtract the sum of $2d-7e+7b-3c$ and $3e-6b-d$.

4. Subtract the sum of $2b+3y-7x-5a$ and $5x-b-4y-a$ from $a+b-3x$.

5. From $3x - 4y + 3z - u$ subtract the sum of $z + x - 8 - 5y$ and $3z + 2x - 2u$.

6. Subtract the sum of $5m - a - 9n$ and $5b + 5n + a - 4m$ from $a + 4b + 6m - 4n$.

7. Subtract $3a - 2d - b - 4c$ from the sum of $4a - 2b + 3c + d$ and $b - 3d + 5a - 2c$.

8. From the sum of $4x + y - 2z$ and $4u + 3y - 7x - 2z$ subtract $4u + 4y - 5z + 5 - 3x$.

9. Subtract the sum of $a^2c - ac^2 + a^2b$, $a^2b - ac^2 - a^2c$, and $ab^2 + a^2c + ac^2$ from $a^2b + a^2c - ac^2$.

10. From the sum of $3x^2 - 2x + 5$ and $4x + 2y - 4$ subtract the sum of $3x - 2x^2 + 3$ and $y - 2x - 2$.

11. Subtract the sum of $xy^2 - xz^2 - x^2z$ and $xz^2 - x^2y - x^2z$ from the sum of $xy^2 - x^2y - x^2z$ and $x^2y - xz^2 - x^2z$.

12. From $4ab - 3ac + 2bc$ subtract the sum of $3bc + bd - ac$, $3ab - 3bd - bc$, and $bd - 2ac - ab$.

13. From the sum of $3xy - 2xz + yz$ and $xz + 5z^2 - xy$ subtract the sum of $4xz - 2yz$ and $4z^2 + xy + yz - 5xz$.

14. From the sum of $2ab - ac + 2bc$ and $2ac - bc - 3ab$ subtract the sum of $3ac - 4bc - ab$ and $2abc - 2ab - 2ac$.

Find the numerical value of the following expressions when $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 0$, $m = \frac{1}{2}$, $n = \frac{1}{3}$:

1. $2ab + 8m - 3n^2 + 2c^2$.
2. $5ad + 8a^2 - 2b - 8m^2$.
3. $4cn + 2d^2 - 7e - 4m^2$.
4. $5ab + 6c^2 - 4dm + 8a^3$.
5. $abc + 2d^2 - 6mn + 2c^2$.
6. $6am + 9a^2 + 2bc - 3m^2$.
7. $bcd - 5e^2 - 4m^3 + 3cn$.
8. $4ad - 6n^2 + 2dm - 2b^2$.
9. $bcd - 9n^2 - 4a^2 + 6am$.
10. $3cd - 3a^2 - 3mn + 4b^2$.

1. What is the interest on x dollars for a years at 5 per cent per annum?

2. What will represent a number in which there are z hundreds, y tens, and x units?

3. How many square feet are there in a piece of paper x yards long and y inches wide?

4. How many square yards are there in the walls of a room x feet square and y feet high?

5. At a cents a square yard, what will it cost to plaster a ceiling x feet long and y feet wide?

6. If a field is $3x$ rods square, how many rods of fence will enclose it and divide it into 4 squares?

7. If x represents an odd number, what will represent the next smaller odd number?

8. What will represent the sum of four consecutive numbers of which x is the largest?

9. How many square yards are there in the ceiling of a room x yards long and y feet wide?

10. What will represent the perimeter of a rectangle that is $5x$ feet long and $3x$ feet wide?

11. B has 3 more cows than A, C has 4 more than B, and D has 5 more than C. How many have all?

12. A has 15 more sheep than B. How many would C have, if he had half as many as A and B together?

13. If A can do a piece of work in x days and B can do it in y days, what part of it can both do in 1 day?

14. At x dollars a week for men and y dollars a week for boys, how much will 6 of each earn in 3 weeks?

15. A bought x sheep at \$3 a head, y lambs at \$2 a head, and had \$45 left. How much money had he at first?

1. A and B together earn $175 a month; A and C, $225; B and C; $240. How much do all earn?

2. A has three times as many cows as B; but if A should sell 6 to B, they would then have the same number. How many cows have both men?

3. Three men raised 1530 bushels of oats. A had three times as many bushels as C, and 248 bushels more than B. How many bushels did B and C raise?

CLEARING EQUATIONS OF FRACTIONS

If equal numbers be multiplied by the same number, or by equal numbers, the products will be equal.

$$\frac{x}{2} - 10 + \frac{x}{4} + 3 = \frac{x}{3} - 5 + \frac{x}{6}.$$

We may clear an equation of fractions by multiplying every term of it by the least common multiple of the denominators. Multiplying each term of this equation by 12, applying cancellation to the fractional terms, we have

$$6x - 120 + 3x + 36 = 4x - 60 + 2x.$$

1. $\frac{x}{2} - 4 + \frac{x}{3} = 3 + \frac{x}{4}.$

2. $\frac{x}{3} - 12 + \frac{2x}{15} = 8 - \frac{x}{5}.$

3. $\frac{x}{5} + 2 + \frac{x}{3} = 4 + \frac{x}{2}.$

4. $\frac{5x}{12} + x - \frac{x}{8} = 38 + \frac{x}{2}$

5. $\frac{x}{2} - 3 + \frac{x}{9} = 2 + \frac{x}{3}.$

6. $\frac{2x}{15} + x - \frac{x}{2} = 18 + \frac{x}{3}.$

7. Half of a number, diminished by 6, is equal to 1 third of the number, increased by 2. Find the number.

8. Three farmers sold 2080 bushels of potatoes. A sold $\frac{1}{4}$ as many bushels as B, and C sold $\frac{3}{5}$ as many bushels as A and B together. How many bushels did B sell?

1. Subtract the sum of $2x - x^2 - y$, $x^3 + 5 - 2x^2$, and $x - x^2 - 4 - y$ from $2x^3 - 4x^2 + 3x$.

2. Subtract the sum of $2b^2 - d^2 + 2c^2$, $a^2 + 6 - c^2$, and $2d^2 - b^2 - 5$ from $2a^2 + 3b^2 + c^2 - d^2$.

3. From the sum of $2x^2y - 3x^2z + xz^2$, $2x^2z - 2xz^2 - x^2y$, and $2xz^2 - xy^2 - x^2z$ subtract $xz^2 - xy - 2x^2z$.

4. Subtract $x^2z - xy - 2xy^2$ from the sum of $x^2y - 2xy + xy^2$, $x^2z - 3x^2y - 5xy^2$, and $3xy + x^2z + 2x^2y$.

5. From the sum of $-a^2b + a^2c - ab^2$ and $ac^2 - 2a^2c + 3a^2b$ subtract the sum of $-a^2c - 2ab^2$ and $2a^2b - 2ac^2 - ab^2$.

6. A's age is 2 thirds of B's, and the sum of their ages is 80 years. Find A's age.

7. A man lost 1 sixth of his money, but he still has $5800 more than he lost. How much has he left?

8. A house and lot cost $4950, the house costing $2\frac{2}{3}$ times as much as the lot. Find the cost of each.

9. A horse and harness cost $260, the horse costing $5\frac{1}{2}$ times as much as the harness. Find the cost of each.

10. A, B, and C together earn two thousand eight hundred dollars. A's salary is $200 more than B's, and $300 less than C's. How much is C's salary?

11. A has 1 fourth as much money as B, and C has $2\frac{2}{5}$ times as much as A and B together. If C's money exceeds B's by $3600, how much have all?

12. A has 1 third as many sheep as B. If A should double his flock and B should sell 80 to C, A and B would then have the same number. How many sheep has B?

13. Frank has 1 fourth as many marbles as John. If John loses 206 and Frank loses 14, they will each have the same number left. How many marbles have both?

MULTIPLICATION

Multiplication is the process of uniting two or more equal quantities. The **Multiplicand** is one of the equal quantities united. The **Multiplier** is the quantity that shows how many equal quantities are united.

$$\begin{array}{cccc} b & -b & b & -b \\ a & a & -a & -a \\ \hline \end{array}$$

The above examples represent all possible cases in multiplication with reference to the signs of the multiplier and multiplicand.

Uniting two b's, the result is $2b$; uniting three b's, the result is $3b$; uniting four b's, the result is $4b$; uniting five b's, the result is $5b$; uniting any number of b's, a representing any number, the result is ab.

Uniting two $-b$'s, the result is $-2b$; uniting three $-b$'s, the result is $-3b$; uniting four $-b$'s, the result is $-4b$; uniting five $-b$'s, the result is $-5b$; uniting any number of $-b$'s, a representing any number, the result is $-ab$.

As has been proved in the first example, the product of b and a is ab. Since the multiplier in the third example is negative, the product must be of the opposite quality from what it would be if the multiplier were positive. Therefore the correct product is $-ab$.

As has been proved in the second example, the product of $-b$ and a is $-ab$. Since the multiplier in the fourth example is negative, the product must be of the opposite quality from what it would be if the multiplier were positive. Therefore the correct product is ab.

It thus appears that when the multiplicand and multiplier have like signs, the product is positive; and when they have unlike signs, the product is negative.

COEFFICIENTS AND EXPONENTS

Since the product must contain all the factors of the multiplier and multiplicand, the numerical factor in the multiplicand must be multiplied by the numerical factor in the multiplier; and if the same letter is used in the two factors, it must appear as a factor as many times in the product

as it is used in multiplier and multiplicand, and this will be indicated by the sum of its exponents.

$$5a \times 3a^2 = 5 \cdot 3 \cdot aaa = 15a^3$$

$$7a^3x^2 \times 2a^2x = 7 \cdot 2 \cdot aaaxxaax = 14a^5x^3$$

$$\begin{array}{r} 3ab \\ 2ab \\ \hline 6a^2b^2 \end{array} \qquad \begin{array}{r} -5a^2c \\ -3ac^2 \\ \hline 15a^3c^3 \end{array} \qquad \begin{array}{r} -6x^3y \\ 4xy^2 \\ \hline -24x^4y^3 \end{array} \qquad \begin{array}{r} 8ax^4 \\ -a^3x \\ \hline -8a^4x^5 \end{array}$$

RULE. — *To the product of the numerical coefficients annex the literal quantities, giving each letter an exponent equal to the sum of the exponents of that letter in both factors. If the signs are unlike, give the product the minus sign.*

$$\textbf{1.}\ \begin{array}{r} ab \\ ab \\ \hline \end{array} \qquad \textbf{2.}\ \begin{array}{r} -4ax^2 \\ 2a^2x \\ \hline \end{array} \qquad \textbf{3.}\ \begin{array}{r} 6xy^2 \\ -ax^2 \\ \hline \end{array} \qquad \textbf{4.}\ \begin{array}{r} -a^2b \\ -b^2c \\ \hline \end{array}$$

$$\textbf{5.}\ \begin{array}{r} ax \\ xy \\ \hline \end{array} \qquad \textbf{6.}\ \begin{array}{r} 3a^3x \\ -5a^2x \\ \hline \end{array} \qquad \textbf{7.}\ \begin{array}{r} -xy^2 \\ 7y^2z \\ \hline \end{array} \qquad \textbf{8.}\ \begin{array}{r} -a^3b \\ -b^2c \\ \hline \end{array}$$

$$\textbf{9.}\ \begin{array}{r} a^2b \\ ab^3 \\ \hline \end{array} \qquad \textbf{10.}\ \begin{array}{r} -4ax^3 \\ 4a^3x \\ \hline \end{array} \qquad \textbf{11.}\ \begin{array}{r} 8ax^2 \\ -4x^2y \\ \hline \end{array} \qquad \textbf{12.}\ \begin{array}{r} -x^2y \\ -x^3y \\ \hline \end{array}$$

$$\textbf{13.}\ \begin{array}{r} ax^2 \\ a^2b \\ \hline \end{array} \qquad \textbf{14.}\ \begin{array}{r} -7ax^2 \\ 2bx^2 \\ \hline \end{array} \qquad \textbf{15.}\ \begin{array}{r} 5a^2x \\ -9x^3y \\ \hline \end{array} \qquad \textbf{16.}\ \begin{array}{r} -a^2x \\ -b^3x \\ \hline \end{array}$$

$$\textbf{17.}\ \begin{array}{r} a^2x \\ xy^3 \\ \hline \end{array} \qquad \textbf{18.}\ \begin{array}{r} -8a^2b \\ x^2y \\ \hline \end{array} \qquad \textbf{19.}\ \begin{array}{r} 6x^3y \\ -a^4x \\ \hline \end{array} \qquad \textbf{20.}\ \begin{array}{r} -a^3x \\ -by^3 \\ \hline \end{array}$$

MULTIPLICATION OF A POLYNOMIAL BY A MONOMIAL

$$\begin{array}{l}
5\,a^3b^2c - 4\,a^2b^3c + 3\,a^2b - 2\,b^2c \\
2\,a^2b^3c \\
\hline
10\,a^5b^5c^2 - 8\,a^4b^6c^2 + 6\,a^4b^4c - 4\,a^2b^5c^2
\end{array}$$

Multiply:

1. $5\,a - ab + 3\,b - 4\,c + 7\,b^2$ by $2\,a^2b$.
2. $b^2c^3 + 2\,b - 2\,c^2 + b^3c^2 - 4\,c$ by $3\,b^2c^2$.
3. $2\,c + 2\,c^2 - a^2c + 3\,a - a^2c^2$ by $4\,ac^3$.
4. $x^2y - 3\,x + 5\,y^2 - x^3y^2 + 3\,y$ by $2\,xy^2$.
5. $a^2b + ab^2 - 5\,a + 3\,b - 4\,b^2$ by $3\,a^2b^2$.
6. $3\,x - ax^2 + 4\,a - a^2x + 2\,x^2$ by $5\,a^3x^2$.

MULTIPLICATION OF A POLYNOMIAL BY A POLYNOMIAL

$$\begin{array}{llllll}
4\,a^3b^2 - & 3\,a^2b^3 + & 2\,ab^4 & + 2\,b^5 & & \\
2\,a^2b \;- & 2\,ab^2 \;+ & b^3 & & & \\
\hline
8\,a^5b^3 - & 6\,a^4b^4 + & 4\,a^3b^5 & + 4\,a^2b^6 & & \\
 - & 8\,a^4b^4 + & 6\,a^3b^5 & - 4\,a^2b^6 & - 4\,ab^7 & \\
 & & 4\,a^3b^5 & - 3\,a^2b^6 & + 2\,ab^7 & + 2\,b^8 \\
\hline
8\,a^5b^3 - & 14\,a^4b^4 + & 14\,a^2b^5 & - 3\,a^2b^6 & - 2\,ab^7 & + 2\,b^8
\end{array}$$

Multiply:

1. $7\,a + 5\,c$ by $3\,a - 4\,c$.
2. $4\,c^2 - 6\,x^3$ by $5\,c - 7\,x$.
3. $3\,c - 5\,y$ by $6\,c + 2\,y$.
4. $5\,a^2 - 4\,b^2$ by $3\,a - 5\,b$.
5. $8\,x - 3\,y$ by $4\,x - 7\,y$.
6. $4\,x^2 - 2\,y^2$ by $5\,x + 3\,y$.
7. $9\,a + 2\,b$ by $2\,a + 9\,b$.
8. $7\,x^3 + 5\,y^2$ by $2\,x - 4\,y$.

Multiply:

1. $3a^3+5a-6$ by $4a^2-5$.
2. $5x^4-3x^3-2x^2$ by x^2-x.
3. $4x^3-2x^2+5x$ by $5x-3$.
4. $3c^2-2c+4$ by c^2-c+1.
5. $4a^2+5a-3$ by $3a^2-a+5$.
6. $3x^3+2x^2-4x+1$ by $5x+4$.
7. $x^3y-x^2y^2-3xy^3$ by $3x^2y-xy^2$.
8. $3ab+2b^2+a^2$ by $2a^2-3ab+b^2$.
9. $5ac-2a^2+3c^2$ by $3a^2+c^2-2ac$.
10. $3x^3-2x+3x^2-5$ by $3+2x^2-4x$.
11. $3xy^2+x^3-3x^2y-y^3$ by $y^2+2xy+x^2$.
12. $3m^2-2m+4-2m^3$ by $2m^2-4-3m$.
13. $2x^4+3x-2x^2-x^3-3$ by $3x+x^2+3$.
14. $3a^3+2a-4a^2+1$ by $2a^3-a-3a^2-4$.
15. $9x^3+2xy^2+3x^2y+4y^3$ by $4x^2+y^2-3xy$.

The **Square** of any quantity is the product obtained by multiplying it by itself.

$$(a+b)^2=(a+b)(a+b)=a^2+2ab+b^2.$$

Since a and b represent any two quantities, $(a+b)^2$ represents the square of the sum of any two quantities. The square of $a+b$ is found by multiplication to be $a^2+2ab+b^2$.

PRINCIPLE. — *The square of the sum of two quantities is the square of the first, plus twice the product of the first and second, plus the square of the second.*

$$(a-b)^2=(a-b)(a-b)=a^2-2\,ab+b^2.$$

Since a represents any quantity and b any less quantity, $(a-b)^2$ represents the square of the difference of any two quantities. The square of $a-b$ is found by multiplication to be $a^2-2\,ab+b^2$.

PRINCIPLE. — *The square of the difference of two quantities is the square of the first, minus twice the product of the first and second, plus the square of the second.*

$$(a+b)(a-b)=a^2-b^2.$$

Since a represents any quantity and b any less quantity, $a+b$ represents the sum and $a-b$ the difference of any two quantities, and $(a+b)(a-b)$ represents the product of the sum and difference of any two quantities. The product is a^2-b^2.

PRINCIPLE. — *The product of the sum and difference of two quantities is the difference of their squares.*

ORAL EXERCISES

1. $(a+x)(a+x)$.

2. $(x+4)(x+4)$.

3. $(a-x)(a-x)$.

4. $(x-3)(x-3)$.

5. $(a+x)(a-x)$.

6. $(x+5)(x-5)$.

7. $(a+2)(a+2)$.

8. $(x-6)(x-6)$.

9. $(a+7)(a-7)$.

10. $(x+8)(x+8)$.

11. $(a-9)(a-9)$.

12. $(x+4)(x-4)$.

13. $(3+a)(3+a)$.

14. $(5-x)(5-x)$.

15. $(6+a)(6-a)$.

16. $(7+x)(7-x)$.

17. $(a+1)(a+1)$.

18. $(x+1)(x-1)$.

19. $(a-5)(a-5)$.

20. $(x+1)(x+1)$.

PRODUCT OF TWO BINOMIALS WITH A COMMON TERM

$$(x+4)(x+3) = x^2 + 7x + 12.$$
$$(x-5)(x-3) = x^2 - 8x + 15.$$
$$(x+7)(x-3) = x^2 + 4x - 21.$$
$$(x+2)(x-9) = x^2 - 7x - 18.$$

The first term of each product is the square of the common term. The second term is the product of the common term and the algebraic sum of the second terms. The third term is the product of the second terms.

ORAL EXERCISES

1. $(x+3)(x+2)$.

2. $(x-5)(x-2)$.

3. $(x+4)(x+1)$.

4. $(x-6)(x-3)$.

5. $(x+5)(x+4)$.

6. $(x-7)(x-2)$.

7. $(x+6)(x+1)$.

8. $(x-6)(x-4)$.

9. $(x+4)(x+2)$.

10. $(a-7)(a-8)$.

11. $(x+7)(x+3)$.

12. $(a-8)(a-1)$.

13. $(x+6)(x+5)$.

14. $(a-7)(a-5)$.

15. $(x+8)(x+4)$.

16. $(a-9)(a-1)$.

17. $(x+9)(x+6)$.

18. $(a-8)(a-3)$.

19. $(x+7)(x-4)$.

20. $(a+2)(a-6)$.

21. $(x-1)(x+6)$.

22. $(a-5)(a+4)$.

23. $(x+6)(x-5)$.

24. $(x+2)(x-3)$.

25. $(a-4)(a+5)$.

26. $(x-7)(x+3)$.

1. A rectangular field $3x$ rods long has a perimeter of $10x$ rods. How wide is the field?

2. If a railway train runs x miles in 3 fourths of an hour, what is the rate per hour?

3. If $2x-5$ represents an odd number, what will represent the next smaller odd number?

4. What will denote the number of acres in a rectangular field x rods long and y rods wide?

5. What will represent the sum of four consecutive odd numbers of which x is the largest?

6. If A can do a piece of work in x days and B in y days, what part of it can both do in 4 days?

7. A man bought x cows at $35 apiece and had $275 left. How much money did he have at first?

8. What will represent the quotient of a number of x hundreds, y tens, and z units, divided by 8?

9. If there are x hundreds, y tens, and z units in a number, what will represent the number?

10. What will represent the sum of five consecutive even numbers of which x is the middle one?

11. What is the age of a man who y years ago was a times the age of a boy whose age was x years?

12. If a park is x rods long and y rods wide, how many times must a man walk around it to travel a miles?

13. If a represents an integer, when does $a+1$ represent an even number? When an odd number?

14. A boy has x quarters, y dimes, and z nickels. Write an expression to denote how many dollars he has.

15. A piece of land is $3x$ rods square. How many rods of fence will enclose it and divide it into 4 squares?

1. One sixth of a man's age added to twice his age equals 78 years. Find the man's age.

2. A boy has $\frac{1}{4}$ as many 5-cent pieces as dimes. If he has $9 in all, how many coins has he?

3. A has twice as many sheep as B and 35 less than C. If all have 635, how many has A?

4. A is $2\frac{2}{3}$ times as old as B, and the sum of their ages is 88 years. Find B's age.

5. The sum of two numbers is 378, and one of them is $3\frac{1}{2}$ times the other. Find the larger number.

6. A man paid 1 fourth of a debt and still owes $90 more than he paid. How much did he pay?

7. One number is $3\frac{1}{3}$ times another number, and their difference is 301. Find the larger number.

8. The perimeter of a rectangle is 132 feet, and the length is $2\frac{2}{3}$ times the width. Find the length.

9. The length of a rectangular field is twice its width, and the perimeter is 144 rods. Find the dimensions.

10. Each of two men has the same number of sheep. If A buys 35 and B sells 15 of his, A will then have twice as many as B. How many have both?

11. A farmer sold 8 ninths of his potatoes, when he found that he had left 868 bushels less than he sold. Find the value of his whole crop at 65 cents a bushel.

12. A man bought two horses, paying $3\frac{3}{5}$ times as much for one as he paid for the other. If the better horse cost $325 more than the other, what did both cost?

13. A farmer sold his corn and wheat for $600, his corn and oats for $400, his wheat and oats for $520. How much did he receive for all of his grain?

DIVISION

Division is the process of finding how many times one quantity is contained in another. Or,

Division is the process of finding one of two factors when their product and the other factor are known.

The **Dividend** is the quantity to be divided. The **Divisor** is the quantity by which another quantity is to be divided. The **Quotient** is the result obtained by division.

Since division is the converse of multiplication, the dividend representing the product, the divisor one factor, and the quotient the other, the rule is deduced from the process of multiplication.

SIGN OF THE QUOTIENT

$$+b \times +a = +ab, \text{ therefore } +ab \div +a = +b$$

$$+b \times -a = -ab, \text{ therefore } -ab \div -a = +b$$

$$-b \times -a = +ab, \text{ therefore } +ab \div -a = -b$$

$$-b \times +a = -ab, \text{ therefore } -ab \div +a = -b$$

These four divisions represent all possible cases in division with reference to the signs. Each quotient is correct, because the quotient and divisor are known to be the factors of the dividend. We see that when dividend and divisor have like signs the quotient is positive; when they have unlike signs the quotient is negative.

EXPONENTS IN THE QUOTIENT

$$a^3c^2x \times ac^3x^2 = a^4c^5x^3, \text{ therefore } a^4c^5x^3 \div ac^3x^2 = a^3c^2x$$

Since the exponent of each letter in the dividend is the sum of the exponents of that letter in both factors, the exponent of each letter in the quotient is found by subtracting the exponent of that letter in the divisor from the exponent of that letter in the dividend.

COEFFICIENT OF THE QUOTIENT

$$5\,a^3 \times 3\,a^2 = 15\,a^5, \text{ therefore } 15\,a^5 \div 3\,a^2 = 5\,a^3$$

The coefficient of the quotient is found by dividing the coefficient of the dividend by the coefficient of the divisor.

$$\begin{array}{r} 2\,a)\underline{8\,a^3} \\ 4\,a^2 \end{array} \qquad \begin{array}{r} -2\,a^2b)\underline{-8\,a^4b^3} \\ 4\,a^2b^2 \end{array} \qquad \begin{array}{r} -3\,a^2b)\underline{12\,a^3b^2c} \\ -4\,abc \end{array}$$

ORAL EXERCISES

1. $4\,ab^2)12\,a^3b^3$. **2.** $3\,a^2b)-15\,a^4b^3$. **3.** $-abc)-6\,ab^2c^3$.

4. $2\,bc^3)16\,b^2c^4$. **5.** $5x^3y)-30\,x^5y^4$. **6.** $-xyz)-5x^2yz^4$.

7. $3\,xy^2)18\,x^4y^4$. **8.** $4\,a^2x)-28\,a^2x^2$. **9.** $-acx)-7\,ac^3x^2$.

10. $5\,cx^3)15\,c^2x^4$. **11.** $2\,b^3c)-18\,b^3c^3$. **12.** $-bxy)-4b^2xy^4$.

13. $6\,b^2)18\,ab^3c^2$. **14.** $7\,x)-21\,a^2x^2y^2$. **15.** $-axy)-3ax^2y^2$.

16. $7\,ac^3)14\,a^3c^3$. **17.** $6\,x^2y)-24\,x^3y^2$. **18.** $-bcx)-2\,b^3cx^3$.

19. $4\,xy^4)16\,x^4y^4$. **20.** $3\,a)-18\,a^3b^3c^2$. **21.** $-abx)-6\,ab^3x^4$.

DIVISION OF A POLYNOMIAL BY A MONOMIAL

$$\begin{array}{r} 4\,ab^2c)\underline{16\,a^2b^4c - 12\,a^3b^2c^2 + 20\,ab^2c^4} \\ 4\,ab^2 \quad - \quad 3\,a^2c \quad + \quad 5\,c^3 \end{array}$$

Divide:

1. $6\,ac^2 - 8\,a^2c + 10\,a^3c^3 - 4\,ac^2$ by $2\,ac$.
2. $8\,x^4y + 4\,x^3y^2 - 16\,x^3y + 4\,x^2y$ by $4\,x^2y$.
3. $6\,b^2c^3 + 10\,b^2c^4 - 8\,b^3c^2 - 2\,b^2c^2$ by $2\,bc^2$.

4. $9a^2b^3 - 3a^3b^4 + 15ab^3 - 6ab^2$ by $3ab^2$.

5. $8x^4y^2 + 4x^3y^2 - 16x^3y^3 + 8x^4y$ by $4x^3y$.

6. $5a^3c^4 + 10a^4c^2 - 5a^3c^3 + 5a^2c^2$ by $5a^2c^2$.

7. $4b^4c^3 + 8b^3c^4d - 16b^3c^3 + 12b^3c^2$ by $4b^3c^2$.

8. $12a^3x^4 + 3a^4x^4 - 6a^2x^3y - 11a^2x^3$ by a^2x^3.

9. $10x^4z^4 - 5x^3yz^5 + 15x^4z^5 - 20x^3z^4$ by $5x^2z^4$.

10. $18a^3c^3 + 12a^3bc^2 - 24a^4c^3 + 6a^3c^2$ by $6a^3c^2$.

11. $32ab^3c^2 - 16b^4c^3d + 16b^3c^4 - 8b^4c^2$ by $8b^2c^2$.

12. $14ab^4c^2 + 28b^4c^3d - 21b^3c^4 + 7b^3c^2$ by $7b^3c^2$.

13. $12b^4x^3y - 18b^3cx^5 + 15b^4x^4 - 6b^2x^3$ by $3b^2x^3$.

14. $15a^4c^3 - 10a^3bc^2d + 25a^3c^3 - 5a^4c^2$ by $5a^3c^2$.

DIVISION OF A POLYNOMIAL BY A POLYNOMIAL

$$
\begin{array}{l|l}
a^4 - a^2b^2 + 2ab^3 - b^4 & a^2 - ab + b^2 \\ \cline{2-2}
\underline{a^4 - a^3b + a^2b^2} & a^2 + ab - b^2 \\
\qquad a^3b - 2a^2b^2 + 2ab^3 & \\
\qquad \underline{a^3b - a^2b^2 + ab^3} & \\
\qquad\qquad - a^2b^2 + ab^3 - b^4 & \\
\qquad\qquad \underline{- a^2b^2 + ab^3 - b^4} &
\end{array}
$$

$$
\begin{array}{l|l}
x^4 + 4y^4 & x^2 - 2xy + 2y^2 \\ \cline{2-2}
\underline{x^4 - 2x^3y + 2x^2y^2} & x^2 + 2xy + 2y^2 \\
\qquad 2x^3y - 2x^2y^2 + 4y^4 & \\
\qquad \underline{2x^3y - 4x^2y^2 + 4xy^3} & \\
\qquad\qquad 2x^2y^2 - 4xy^3 + 4y^4 & \\
\qquad\qquad \underline{2x^2y^2 - 4xy^3 + 4y^4} &
\end{array}
$$

Divide:

1. $14a + a^2 - 72$ by $a - 4$.
2. $x^2 - 13x + 12$ by $x - 1$.
3. $15x^3 + x^6 + 56$ by $7 + x^3$.
4. $a^4 + 15a^2 + 36$ by $a^2 + 3$.
5. $y^4 - 19y^2 + 48$ by $y^2 - 3$.
6. $b^6 - 15b^3 + 56$ by $b^3 - 7$.
7. $a^6 + 50a^3 + 96$ by $a^3 + 2$.
8. $y^4 + 47y^2 - 48$ by $y^2 - 1$.
9. $a^4 + ac^3 + a^3c + c^4$ by $a + c$.
10. $b^3 + b^2x + bx^2 + x^3$ by $b + x$.
11. $a^3 - ax^2 - a^2x + x^3$ by $a - x$.
12. $a^4 + c^4 + a^2c^2$ by $a^2 + c^2 + ac$.
13. $ac^3 - abc - bc^2 + b^2$ by $ac - b$.
14. $x^3 + 3xy^2 - 3x^2y - y^3$ by $x - y$.
15. $x^2 - y^2 + z^2 - 2xz$ by $x + y - z$.
16. $a^2 + 2xy - y^2 - x^2$ by $a + x - y$.
17. $32x^3 - 6x + 1$ by $8x^2 + 2x - 1$.
18. $4a^4 - 6a^2 - 8a^3 + 24$ by $2a - 4$.
19. $4b^3 + 4ab^2 - 3a^2b - 3a^3$ by $b + a$.
20. $16a^4 + 81x^4 - 72a^2x^2$ by $2a - 3x$.
21. $25x^5 - 8x - 2x^2 - x^3$ by $5x^2 - 4x$.
22. $x^3 + 1$ by $x + 1$.
23. $64a^3 + 8b^3$ by $4a + 2b$.
24. $a^5 + 1$ by $a + 1$.
25. $27x^3 + 8y^3$ by $3x + 2y$.
26. $x^3 + 8$ by $x + 2$.
27. $36b^2 - 81c^2$ by $6b - 9c$.
28. $8 - x^3$ by $2 - x$.
29. $25x^2 - 16y^2$ by $5x + 4y$.
30. $a^4 - b^4$ by $a - b$.
31. $16a^4 - 81b^4$ by $2a - 3b$.
32. $x^5 + y^5$ by $x + y$.
33. $81x^4 - 16y^4$ by $3x + 2y$.
34. $x^6 + y^6$ by $x^2 + y^2$.
35. $16a^2 - 64b^2$ by $4a - 8b$.
36. $a^8 - y^8$ by $a^2 - y^2$.
37. $49x^2 - 25y^2$ by $7x + 5y$.

Find the numerical value of the following expressions when $a = 10$, $b = 8$, $c = 5$, $d = 4$, $e = 3$, and $x = 1$:

1. $a(b + c)$.
2. $(e + x)cd^2$.
3. $e^3x^2(a - b)^3$.
4. $(a - e)d^2$.
5. $abx(a + c)$.
6. $(a + 2d)cx^2$.
7. $(b - c)ax^2$.
8. $(a - x)d^2e^2$.
9. $(x^3 + c^2)2ce^2$.
10. $dx^2(c + e)$.
11. $(a - c)5ex$.
12. $2dex(c - d)^2$.
13. $a^2e^2(a - b)$.
14. $2c^2x^3(c + x)$.
15. $(a^2 - b^2)3ce^2x^2$.

ORAL EXERCISES

1. $(x + 1)(x + 1)$.
2. $(a + 5)(a - 1)$.
3. $(x - 4)(x - 4)$.
4. $(a + 7)(a - 8)$.
5. $(x + 2)(x - 2)$.
6. $(x + 9)(x + 5)$.
7. $(x - 1)(x - 1)$.
8. $(x - 9)(x - 4)$.
9. $(x + 3)(x + 3)$.
10. $(a - 6)(a + 7)$.
11. $(x - 8)(x + 5)$.
12. $(x + 1)(x - 1)$.
13. $(x + 5)(x + 5)$.
14. $(a + 4)(a - 2)$.
15. $(a - 2)(a - 2)$.
16. $(x + 5)(x - 9)$.
17. $(x + 3)(x - 3)$.
18. $(a - 3)(a + 4)$.
19. $(x - 3)(x - 3)$.
20. $(x - 8)(x + 6)$.
21. $(x + 6)(x - 2)$.
22. $(a + 6)(a + 6)$.
23. $(x + 4)(x - 9)$.
24. $(x + 6)(x - 6)$.
25. $(a - 5)(a - 5)$.
26. $(x + 9)(x + 2)$.
27. $(x - 9)(x + 2)$.
28. $(a + 7)(a + 7)$.
29. $(x - 7)(x - 4)$.
30. $(x + 9)(x - 9)$.

1. A is three times as old as B, but 10 years was 5 times as old as B. Find A's age now.

2. A farmer bought 3 times as many sheep already had and then had 248. How many did he

3. A has twice as much money as B, and B has as much as C. If all have $455, how much has C?

4. How many boys are there in a school of 675 p if twice the number of boys equals the number of gir

5. A has $1\frac{1}{2}$ times as many sheep as B, and C ha times as many as B. If all have 336, how many has (

6. Twice a number, increased by 96, is equal to times the number, diminished by 29. Find the num

7. A lady spent 1 eighth of her money for a dress, wh she found that she had left $144 more than she had spe How much did she have left?

8. The sum of the ages of A, B, and C is one hundre forty-eight years. A is three times as old as C, and eigh years younger than B. Find the age of B.

9. D is 4 years older than C; C is 3 years older than B; B is 2 years older than A. If they live 5 years, the sum of their ages will be 92 years. Find D's age.

10. A man gave $110 to his five sons, each of four of them receiving $6 more than his next younger brother. How much did the oldest son receive?

11. A farmer sold 20 lambs and 50 sheep for $240. He received twice as much per head for the sheep as for the lambs. How much did he receive for the 50 sheep?

12. The sum of three numbers is 142. The second exceeds the first by 9, and the third is 17 less than the second. Find the sum of the second and third numbers.

WERS

ED ARITHMETIC

GRADE

$14\frac{7}{24}$.
$25\frac{9}{40}$.
$25\frac{11}{12}$.
$15\frac{7}{12}$.
$18\frac{2}{15}$.

age 704.

$\frac{1}{14}$.
$\frac{9}{20}$.
$\frac{5}{12}$.

5.

8. $31\frac{1}{2}$.
9. $69\frac{2}{9}$.
1. $251\frac{3}{4}$.
2. $291\frac{1}{2}$.
3. $294\frac{2}{3}$.
4. 624.
5. $413\frac{2}{5}$.
6. 357.
7. 328.
8. $737\frac{1}{2}$.
9. $408\frac{1}{4}$.

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1. $45\frac{1}{2}$.
2. $64\frac{5}{12}$.
3. $57\frac{4}{9}$.
4. $43\frac{2}{5}$.
5. $36\frac{3}{7}$.
6. $29\frac{1}{3}$.
7. $56\frac{3}{8}$.
8. $34\frac{2}{9}$.
9. $27\frac{19}{65}$.
1. $2\frac{1}{2}$.
2. $1\frac{9}{10}$.
3. $2\frac{2}{5}$.
4. $2\frac{2}{7}$.
5. 4.
6. $2\frac{1}{2}$.
7. $1\frac{1}{2}$.
8. $3\frac{31}{35}$.
9. $1\frac{1}{4}$.

1. Two thirds of a number added to 3 fourths of the number equals 204. Find the number.

2. A is 4 times as old as his son, and 5 years ago he was 7 times as old. Find the father's age.

3. Two horses cost $ 560. The difference in cost was $1\frac{1}{2}$ times the cost of the first. Find the cost of each.

4. A man bought 45 sheep and had $ 30 left. If he had bought 60 at that price, he would have needed $15 more to pay for them. How much money did he have?

5. I bought silk at $ 3 a yard and had $ 12 left. Twice as many yards at $ 2 a yard would have cost $ 2 more than I had. Find the cost of the silk bought.

6. The sum of the ages of mother and daughter is 60 years, and the difference between their ages is three times the daughter's age. Find the mother's age.

7. A man left $\frac{2}{5}$ of his estate to his wife and $\frac{1}{4}$ of it to his daughter. The wife received $4200 more than the daughter. How much did the daughter receive?

8. A horse, carriage, and harness cost $292. The horse cost $90 more than the harness, and the carriage cost $23 less than the horse. Find the cost of the horse.

9. Six boys and fifteen men earn two hundred sixty-four dollars a week. If each man earns four times as much as each boy, how much do the boys earn per week?

10. A man bought 2 carriages for $305. At the same prices, 3 of the poorer ones would cost $15 more than 2 of the better ones. Find the cost of the better carriage.

ANSWERS

COMPLETE GRADED ARITHMETIC

EIGHTH GRADE

Page 695.

1. 2^4, 17.
2. 2, 3, 5, 11.
3. 5^3, 13.
4. 2^4, 3^2, 13.
5. 3^2, 7, 11.
6. 2^2, 11^2.
7. 2, 3, 5, 7, 11.
8. 2^5, 41.
9. 3^2, 7^2.
10. 3^2, 5, 13.
11. 3^2, 7, 19.
12. 5^2, 11^2.

1. 12.
2. $7\frac{1}{3}$.
3. $5\frac{1}{4}$.
4. $3\frac{17}{21}$.
5. \$967.50.
6. 60.
7. 125.

Page 697.

1. 960.
2. 1260.
3. 720.
4. 6650.
5. 6160.
6. 770.
7. 720.
8. 5616.
9. 630.
10. 5670.
11. 240.
12. 1728.
13. 2520.
14. 83,160.
15. 756.
16. 2240.
17. 420.

Page 698.

1. 13 ; 11 ;
   1 ; 1.
2. 15 ; 32 ; 4.

1. 192.
2. 13.
3. 19.
4. 126.
5. 4.
6. 87.

Page 699.

1. 75,712.
2. 180.
3. 1584 ft.
4. \$6048.
5. \$240.
6. 24.
7. 67.
8. 34.
9. 252.
10. 265.
11. 47.

Page 703.

1. $21\frac{11}{12}$.
2. $13\frac{7}{20}$.
3. $16\frac{1}{20}$.
4. $21\frac{1}{4}$.
5. $14\frac{7}{24}$.
6. $25\frac{9}{40}$.
7. $25\frac{11}{12}$.
8. $15\frac{7}{12}$.
9. $18\frac{2}{15}$.

Page 704.

1. $4\frac{11}{14}$.
2. $4\frac{9}{20}$.
3. $5\frac{5}{12}$.
4. $4\frac{17}{24}$.
5. $4\frac{17}{18}$.
6. $5\frac{7}{20}$.
7. $3\frac{5}{6}$.
8. $3\frac{7}{12}$.
9. $5\frac{11}{15}$.
10. $8\frac{5}{24}$.
11. $4\frac{5}{12}$.
12. $11\frac{37}{40}$.
13. $10\frac{11}{12}$.
14. $3\frac{5}{18}$.
15. $8\frac{11}{12}$.

Page 705.

1. $30\frac{2}{3}$.
2. 70.
3. $37\frac{4}{5}$.
4. $84\frac{3}{8}$.
5. $37\frac{5}{7}$.
6. 61.
7. $26\frac{1}{2}$.
8. $31\frac{1}{2}$.
9. $69\frac{2}{9}$.

1. $251\frac{3}{4}$.
2. $291\frac{1}{2}$.
3. $294\frac{2}{3}$.
4. 624.
5. $413\frac{2}{5}$.
6. 357.
7. 328.
8. $737\frac{1}{2}$.
9. $408\frac{1}{4}$.

Page 707.

1. $45\frac{1}{2}$.
2. $64\frac{5}{12}$.
3. $57\frac{4}{9}$.
4. $43\frac{2}{5}$.
5. $36\frac{3}{7}$.
6. $29\frac{1}{3}$.
7. $56\frac{3}{8}$.
8. $34\frac{2}{9}$.
9. $27\frac{19}{63}$.

1. $2\frac{1}{2}$.
2. $1\frac{9}{10}$.
3. $2\frac{2}{5}$.
4. $2\frac{2}{7}$.
5. 4.
6. $2\frac{1}{2}$.
7. $1\frac{1}{2}$.
8. $3\frac{31}{35}$.
9. $1\frac{1}{4}$.

Page 708.

1. 660.
2. $8\frac{5}{24}$.
3. $16\frac{7}{18}$.
4. $5\frac{43}{60}$.
5. $1312.
6. $10\frac{5}{12}$.
7. $3\frac{13}{15}$.
8. $8\frac{11}{18}$.
9. 40.
10. $\frac{8}{15}$.
11. $2\frac{11}{12}$.
12. $2\frac{2}{3}$.
13. $50\frac{5}{8}$.
14. $7\frac{3}{8}$.
15. $35\frac{3}{4}$.
16. $31\frac{3}{8}$.
17. $5\frac{3}{8}$.
18. $3\frac{1}{12}$.
19. $1748\frac{4}{7}$.
20. $1\frac{1}{2}$.
21. 190.
22. $2\frac{6}{17}$.
23. $4368\frac{4}{5}$.
24. $8\frac{1}{12}$.
25. $560.
26. $10\frac{1}{3}$.
27. $3\frac{7}{8}$.
28. $95.
29. 840 bu.
30. $599.55.

Page 710.

1. $\frac{9}{50}$; $\frac{17}{40}$; $\frac{1}{40}$; $\frac{1}{80}$.
2. $\frac{13}{50}$; $\frac{69}{200}$; $\frac{9}{125}$; $\frac{3}{400}$.
3. $\frac{17}{20}$; $\frac{7}{40}$; $\frac{1}{25}$; $\frac{1}{16}$.
1. $.23\frac{1}{3}$; .475; .216; .00225.
2. .225; $.22\frac{2}{3}$; .1275; .0888.
3. $.11\frac{2}{9}$; .1625; $.123\frac{1}{3}$; $.0263\frac{1}{3}$.

Page 711.

1. .1875; .489 +; .489 +; .013 +.
2. .257 +; .731 +; .351 +; .0065 +.
1. 733.71.
2. 949.625.
3. 429.847.
1. 3.165.
2. 25.85.
3. 25.615.
4. 33.55.
5. 53.56.
6. 35.455.
7. 51.795.
8. 79.625.
9. 49.574.

Page 712.

1. 12110.8.
2. 259.862.
3. 2639.48.
4. 2571.45.
5. 32575.2.
6. 4758.4.
7. 372.42.
8. 4164.
9. 341.1.
1. 550.08.
2. 40.554.
3. 6.6199.
4. 182.6.
5. 53.712.
6. 7.02.

Page 713.

1. 47.236.
2. 1.29024.
3. 6.7656.
4. 33.714.
5. 9.45.
6. 3.06.
7. 6.825.
8. 42.812.
9. 8.092.
1. 565.88.
2. 1513.525.
3. 1546.857.
4. 33.12.
5. 342.9328.
6. 251.475.
7. 7.35.
8. 68.9052.
9. 65.553.
1. 47.05.
2. 80.6.
3. .08562.
4. .1993 +.
5. 6.04.
6. .435.
7. 70.8.
8. .0506.
9. 8.05.

Page 714.

1. 43.75.
2. 3.75.
3. 37.5.
4. 1.725.
5. 8.4.
6. 57.5.
7. 1.086.
8. 21.6.
9. 4.008.
1. 204.
2. $7140.
3. 840.
4. $29.98.
5. $155.
6. $43.40.

Page 716.

1. $81.90; $122.85; $92.14.
2. $80.94; $107.92; $97.80.
3. $103.68; $90.72; $96.48.
4. $24.83; $28.96; $33.10; $24.
5. $59.54; $75.89; $86.40.

Page 717.

6. 9475.
7. 1480.
8. $25.54.
9. 17,350.
10. $217.26; $319.50; $357.84.
11. $67.08.
12. $103.05; $127.10; $130.53.
13. 17,200.
14. $78.98; $91.55; $107.70.
15. $137.63; $178; $196.35.
16. $102.38.
17. $119.49.
18. 2200 lb.
19. 8965.
20. $25.16.

Page 718.

1. 6375.
2. 3000.
3. $71.04.
4. $246.40.
5. 37,800.

Page 720.

1. $29\frac{5}{9}$; $30\frac{1}{2}$.
2. $40\frac{3}{4}$; $41\frac{11}{18}$.
3. $62\frac{2}{9}$; 67.
4. $758.16.
5. 5084.
6. $482.02.
7. $127.04.

Page 721.

1. 410.45.
2. 2633.4.
3. 1615.95.

4. 3770.55.
5. 9482.44.
6. 4176.4.

Page 722.

1. 1389.31.
2. 925.2.
3. 364.8.
4. 482.4.
5. 1070.1.
6. 603.
7. 606.2.

Page 723.

1. 539; 918; $2214; $204; $45.60.
2. 141.
3. $40,000; $112,500; $97,500.

Page 724.

1. 3875; 895; 4200.
2. $513.
3. 850; 1800; 7080.
4. 450; 4060; $648.
5. $22.24.
6. $147.
7. $600.
8. 776.
9. 495.
10. 888; 4850; 48,000.
11. 2916.

Page 725.

1. 15%; 35%; 8%; $62\frac{1}{2}$%.
2. 24%; 106%; $\frac{1}{4}$%; $7\frac{1}{3}$%.
3. 65%.
4. 64%.
5. $833\frac{1}{3}$%.
6. $16\frac{1}{2}$%.
7. 68%.
8. 54%.
9. $13\frac{4}{7}$%.

Page 726.

1. $125.58.
2. 32%.
3. $2123.
4. 44%.
5. 341.
6. 38,300.
7. 294.
8. $825.
9. $76,800.
10. $57\frac{1}{2}$%.
11. $3027.60.
12. $6800.
13. $135,000.

Page 728.

1. $251.10.
2. $7500; $1350.
3. $13\frac{1}{3}$%.
4. $.05.
5. 15%.
6. $92.40.
7. $.04.
8. $7475.
9. $.75.
10. $665.
11. $3328.
12. $413.14.
13. $228.75.
14. $23\frac{1}{13}$%.
15. $18\frac{1}{3}$%.

Page 729.

1. $98.
2. $160.
3. $2900.
4. $33\frac{1}{3}$%.
5. $4081.
6. $236.50.
7. $903.
8. $4\frac{1}{3}$%.
9. $60.
10. $200
11. $1540.
12. $250; $600.

Page 731.

1. $2748.49.
2. $60.30.
3. $1181.25.
4. $4\frac{1}{2}$%.
5. $1031.40.
6. 5%.
7. $339.
8. 6%.
9. $3068.50.
10. 5%.
11. $2732.20.
12. $97.50.
13. $121.50.
14. $17.20.
15. $40,320.

Page 732.

1. $746.70.
2. $13,446.40.
3. $35.60.
4. 5%.
5. $822.50.
6. 560.
7. $67.
8. $28.75.
9. $450.
10. $39.60; 1650.

Page 733.

1. $33.75.
2. $4800.
3. $8400.
4. $600.
5. $7000.
6. $600.
7. $24,375.

Page 734.

1. $52.50.
2. $14,000.
3. $60,000.
4. $\frac{3}{4}$%.
5. $88.
6. $4800.
7. $3371.25.
8. $4300.

Page 738.

1. $85.85.
2. $1548.40.
3. $372.35.
4. $265.74.
5. $57.48.
6. $1032.50.
7. $40.86.
8. $110.40.
9. $476.43.
10. $213.33.
11. $1666.
12. $135.90.
13. $1475.60.
14. $60.15.
15. $761.01.
16. $901.90.
17. $47.83.

Page 739.

1. $15.20.
2. $4390.97.
3. $3331.79.
4. $498.18.
5. $840.70.
6. $29.24.
7. $3528.
8. $67.52.
9. $1914.11.

Page 740.

1. $17.69.
2. $143.26.
3. $308.22.
4. $1678.85.
5. $80.89.
6. $637.96.
7. $11.89.
8. $544.66.
9. $1393.97.
10. $1823.05.
11. $346.72.
12. $943.24.
13. $9740.03.
14. $753.01.
15. $1055.70.
16. $2245.59.

Page 742.

1. $6012.
2. $4980.
3. $1578.21.
4. $1272.95.
5. $4568.38.
6. $6898.75.
7. $7104.20.

Page 743.

1. $3359.17.
2. $1164.43.
3. $3889.56.
4. $5081.15.
5. $4368.72.
1. $2073.60.
2. $2586.47.
3. $1474.45.
4. $7650.15.

Page 744.

1. $1402.28.
2. $1279.59.

Page 745.

1. 4%.
2. 6%.
3. 5%.
4. 7$\frac{1}{2}$%.
5. 7%.
1. 9 mo. 8 da.
2. 9 yr. 5 mo.
3. 7 mo. 21 da.
4. 3 yr. 3 mo. 2 da.

Page 747.

1. $1493.35.
2. $1407.82.
3. $3765.08.
4. $2371.06.
5. $1019.39.

Page 748.

1. $616.31.
2. 66$\frac{1}{2}$%.
3. $451.87.
4. 15%.
5. $97.
6. $28.72.
7. $1050.
8. $93.01.
9. $142.75.
10. $567.
11. 16$\frac{2}{3}$%.

Page 749.

1. $36.99.
2. $4267.61.
3. $935.31.
4. $927.39.
5. $72.13.
6. $4586.75.

Page 751.

1. $467.87.
2. $737.35.

Page 752.

1. $469.46.
2. $691.60.
3. $10.08;
   $1249.92.
4. $6.50;
   $648.92.
5. $28.26;
   $906.74.
6. $18.71;
   $1607.96.
1. $851.49.
2. $808.82.

Page 755.

1. $879.38.
2. $646.40.

Page 756.

1. $856.38.
2. $554.16.
3. $1697.69.
4. $576.44.
5. $2661.63.
6. $8067.23.

Page 758.

1. $9243.75.
2. $5206.25.
3. $1000.
4. 48 shares;
   $384.
5. 75 shares.
6. $725.
7. $795.
8. $637.50.
9. $36,437.50.

Page 759.

1. $400.
2. $840.
3. $20,850.
4. $14,472.50.
5. $675.
6. $19,563.75.
7. $15,712.50.
8. $25,520.

Page 760.

1. 7$\frac{1}{7}$%.
2. 8%.
3. 5%.
4. 4%.
5. $252,000.
6. $14,175.
7. $600 dim.
8. $1440.
9. $70 inc.
10. $2277.

Page 761.

1. $750.
2. A, $4400;
   B, $2800.
3. $2568.

Page 762.

1. A, $7200;
   B, $7200;
   C, $3600.
2. $1075.
3. $3522.73;
   $4227.27.
4. A, $5000;
   B, $4000;
   C, $7000.

Page 763.

1. A, $1608;
   B, $2010;
   C, $2814.
2. A, $2570;
   B, $2570;
   C, $1285.
3. A, $4441.50;
   B, $4441.50.
4. A, $3496;
   B, $3952.
5. $12,400;
   Oct. 1st.
6. A, $2898;
   B, $2184;
   C, $3360.
7. A, $2982;
   B, $2840;
   C, $1136.

Page 766.

1. 16.
2. 12$\frac{1}{2}$.
3. 12$\frac{1}{2}$.
4. 352.

Page 767.

5. 8$\frac{1}{4}$.
6. 7$\frac{1}{4}$.
7. $4000.
8. 6$\frac{1}{4}$.
9. $3.76.
10. 420.
11. $39.20.

Page 768.

1. 27.
2. 1674.
3. 24.
4. 44.
5. 12.
6. 8.
7. $70.35.
8. 20.
9. 22$\frac{11}{12}$.
10. 7$\frac{1}{2}$.
11. 15.
12. 6 ft.
13. 14.

Page 769.

14. 2.
15. 7.
16. 344.
17. 10.
18. 18.
19. \$3.80.
20. \$1200.
21. 10.
22. 36.
23. 27.
24. 16.
25. $8\frac{3}{4}$.

Page 770.

1. 784.
2. 103,823.
3. 3364.
4. 50,625.
5. 274,625.
6. 7569.
7. 405,224.
8. 193,536.
1. $\frac{125}{512}$.
2. $\frac{144}{289}$.
3. $72\frac{1}{4}$.
4. 3.375.
5. $\frac{343}{729}$.
6. $\frac{256}{625}$.
7. $49\frac{8}{27}$.
8. 983.4496.

Page 771.

1. 137.
2. 901.
3. 3706.
4. 4940.
5. 307.
6. 508.
7. 5819.
8. 7070.
9. 461.
10. 630.
11. 4028.
12. 5991.

Page 772.

1. 1.0039+.
2. .6324+.
3. .09487+.
4. 30.0699+.
5. 8.0031+.
6. .8366+.
7. .1581+.
8. 8.9481+.
9. 11.7770+.
10. .9487+.
11. 1.0049+.
12. 1.8718+.
1. 2.3452+.
2. .9128+.
3. 2.1602+.
4. $2\frac{1}{2}$.
5. 2.9580+.
6. .7558+.
7. $1\frac{2}{3}$.
8. 2.2990+
9. $2\frac{2}{3}$.
10. .8819+.
11. 2.7928+.
12. 3.1358+.

Page 774.

1. 509.
2. 243.
3. 586.
4. 672.
5. 301.
6. 681.
7. 708.
8. 352.
9. 805.
1. 2.094+.
2. 1.638+.
3. .646+.
4. 738.
5. 3.805+.
6. 4.061+.
7. .851+.
8. 894.
9. 9.969+.
10. 5.902+.
11. .777+.
12. 906.

Page 775.

1. 49° 26′ 40″.
2. 111° 8′ 23″.
3. 74° 19′ 32″.

Page 776.

1. 10 : 5 A.M.
2. 1 : 55 P.M.
3. 10 A.M.
1. 76°.
2. 33° 48′.
3. 125° 10′ E.
4. 92° 5′.
5. 2° 20′ 12″ E.

Page 777.

1. 4 hr. 21 min. 21 sec.
2. 5 hr. 51 min.
3. 4 hr. 49 min. $13\frac{3}{5}$ sec.
4. 146° 15′.
5. 3 hr. 53 min. 23 sec.
6. 12 : 3 : $20\frac{1}{3}$ P.M.
7. 2 : 10 P.M.
8. 122° 26′ 15″ W.
9. 2 : 34 : $2\frac{2}{3}$ P.M.
10. 2° 20′ 15″ E.

Page 778.

11. 122° 40′.
12. 8 hr. 2 min. 41 sec.
13. 55° 5′ E.
14. 6 : 17 : 56 P.M.
15. 6 hr. 21 min. 4 sec.
16. 6 : 20 A.M.
17. 2 : 26 : $1\frac{1}{5}$ P.M.
18. 8 : 34 : 29 A.M.
19. 1 hr. 17 min. $20\frac{2}{15}$ sec.
20. 10 : 3 : $14\frac{2}{3}$ A.M.
21. 3 : 51 : 49 P.M.
22. 84° 58′ W.

Page 780.

1. 463.95^{m}; 4.6395^{Hm}; 4639.5^{dm}; $.46395^{Km}$; 46.395^{Dm}.
2. 34000^{m}; 3400000^{cm}; 3400^{Dm}; 340000^{dm}; 340^{Hm}.
3. 831.058^{Km}.
4. 1506.89^{Hm}.
5. \$37.44.
6. \$23.52.
7. 2400.
8. 800.

Page 781.

1. 3360.
2. 4.116.
3. 3000^{dm}.
4. 3150 ; 31.5.
5. 387.
6. 140^{m}.
7. 26.972.
8. 75^{Dm}.
9. 80.
10. \$8160.
11. 96,750.
12. \$243.60.
13. 65^{Dm}.

Page 782.

1. 82.8.
2. 100^{dm}.
3. 9.6^{m}.
4. \$224.64.
5. $3120^{cu\ dm}$.

Page 783.

1. 6506.54^{l}; 42800^{l}; 428^{Hl}.
2. 468^{Hl}.
3. $35,280^{l}$.
4. $3^{m}\ 8^{dm}$.
5. 13.3632^{Hl}.
6. $8\frac{1}{3}^{dm}$.
7. 1105.92^{Hl}.
8. 3^{dm}.

Page 784.

1. $54,872^{Kg}$.
2. 295.65^{Kg}.

3. 1,500,000$^{\text{dg}}$.
4. $2\frac{1}{2}^{\text{dm}}$.
5. 14$^{\text{m}}$.
6. 25.19 hr.
7. 803.6$^{\text{Hl}}$.
8. 180,000.
9. 2$^{\text{m}}$.
10. 5280$^{\text{Dg}}$.
11. 26.31 hr.
12. 108$^{\text{Kg}}$.
13. 5292$^{\text{l}}$; 52.92$^{\text{Hl}}$; 5292$^{\text{Kg}}$.

Page 786.

1. 5192.
2. 33.
3. 48.415.

Page 787.

1. 36 rd.
2. 16.8 ch.
3. 195 ft.
1. 40 rd.
2. 101.16 rd.
3. 15.58 ft.
4. 21 ft.
5. 15 rd.
6. 37.9 ft.
7. $400.
8. 340.

Page 788.

1. 7 A.
2. 22.5 A.
3. 30 rd.
4. 24 ft.
5. $406.

Page 789.

1. 24.4 in.
2. 20.7 in.
3. 22.5 rd.
4. 8.9 ft.
5. 25 in.
6. 42.4 in.
7. 170.8 rd.
8. 64.7 ft.

Page 790.

1. 21.2 in.
2. 12.5 rd.
3. 35.3 rd.
4. 9.5 in.
5. 56.5 rd.
6. 18.3 ft.
7. 42 ft.
8. 272.6 ft.
9. 7.2 ft.
10. 64.03 ft.
11. $280.

Page 791.

1. 154.1 sq. ft.
2. 84.8 sq. ft.
3. 35.3 ft.
4. 110.6 sq. ft.
5. 249.4 sq. ft.
6. 64.03 ft.
7. 216.
8. 15 rd.; 300 sq. rd.
9. 19.2 ft.
10. 136.7 sq. ft.
11. 15 ft.
12. 16.5 ft.

Page 792.

1. 13,125 sq. ft.
2. 4080 sq. ft.
3. 240 sq. rd.
4. 17.5 ch.
1. 6192 sq. ft.
2. 132 sq. ft.
3. 2850 sq. rd.
4. $1845.
5. $16\frac{4}{5}$ A.

Page 793.

1. 476 sq. in.
2. 864 sq. ft.
3. $65\frac{4}{9}$.
4. 135 sq. ft.
5. $1250.
6. 1320 sq. ft.
7. 3 A.

Page 794.

1. 56.5488 ft.; 213.6288 ft.
2. 8.27 rd.; 4.135 rd.
3. 87.9648 ft.; 87.9648 yd.
4. 5.092 rd.; 2.546 rd.
5. 179.0712 in.
6. 43.9824 in.
7. 28.2744 in.

Page 795.

1. 12.5664 sq. ft.
2. 38.4846 sq. ft.
3. 346.5 sq. ft.
4. 8144 sq. rd.
5. 11,192 sq. ft.
6. 904.78 sq. ft.
7. 6361.74 sq. ft.
8. 3341.5 sq. ft.
9. 150.7968 ft.
10. 1150.5 sq. ft.

Page 796.

1. 7.07 rd.
2. 5 ft.
3. 31.6987 rd.
1. 9.8 in.
2. 12.7 ft.
3. 36.695.
4. 206.93 sq. ft.
5. 15.5 in.
6. 612.5 sq. ft.

Page 797.

1. 67.5 sq. ft.
2. 113.0976 sq. ft.
3. 672 sq. in.
4. 146.0844 sq. ft.
5. 1642.18 sq. in.

Page 798.

1. 1809.56.
2. 31.34.
3. 2256.3.
4. 9 cu. ft.
5. 119.38 sq. ft.
6. 5980.8.
7. 226.195 sq. ft.

Page 799.

1. 672 sq. in.
2. 136 sq. ft.
3. 448 sq. in.
4. 73.0422 sq. ft.
5. 24 cu. ft.
6. 927.68 cu. in.
7. 175 cu. ft.
8. 183.48 sq. ft.
9. 208.13 sq. ft.
10. 6.92 cu. ft.
11. $70.
12. $17\frac{2}{3}$.

Page 800.

1. 804.249 sq. in.; 2463.014 sq. in.
2. 1809.56 sq. in.
3. 904.78 cu. in.; 3053.63 cu. in.
4. 105.74 gal.
5. 410.4 sq. in.; 4071.51 sq. in.
6. 11.04 cu. ft.; 8 cu. ft.
7. 57.08 rd.
8. 17.6 in.
9. 19.09 in.

Page 801.

1. 2744 cu. in.
2. 6.9 ft.
3. 249.4 sq. ft.
4. 141.37 in.
5. 211.8 ft.
6. 4071.5 cu. in.
7. $42\frac{2}{3}$ cu. ft.

8. 20.5 ft.
9. 9600.
10. 98 sq. ft.
11. $6\frac{3}{40}$ A.
12. 61.35 sq. ft.
13. $24\frac{23}{40}$ A.
14. 3960 sq. ft.

Page 804.

1. 207.
2. 13.
3. 530.
4. 112.

Page 805.

1. 74.
2. 125.
3. $3275.
4. $14,000.
5. A, 170; B, 340; C, 85.
6. $7500.
7. 27.
8. 1040.
9. 250.
10. $1500.
11. 100.
12. 45.

Page 807.

1. 77.
2. 22.
3. 54.
4. 47.
5. 26.
6. 35.
7. 107.
8. 53.
9. 54.
10. 13.

Page 809.

1. $x - y - 2z$.
2. $3b - 3c$.
3. $ac - bc$.
4. $a^2b - ab^2 + 2a^2b^2$.
5. $xy^2 + x^2y - x^2y^2$.
6. $5a^3 - 6a$.

Page 812.

1. 57; 91.
2. 222.
3. 18 ft.; 54 ft.
4. 135 ft.
5. 34.
6. 196.
7. $14,000.
8. $225.
9. $2400.
10. 162.
11. $32.
12. 1200.
13. 56.

Page 814.

1. $ab - 5ac - xy$.
2. $4ay - 13xy - xz$.
3. $3ax - 4az + 8$.
4. $x^2y + xy^2 + 5xz - 7$.
5. $61x^3 + x^2y + 13xy - 8$.
6. $ab^2 + 61b^3 + 7$.
7. $bc^2 - 5b^2c - 9$.
8. $a^2x - 15a^2y - 56 - z$.
9. $19x^3 + 86y^2 - xyz + 8$.
1. $-a - 4b - c - 4$.
2. $y + z + u + 8$.
3. $2c$.
4. $7a - x + y$.

Page 815.

5. $y - z + u + 8$.
6. $a - b + 5m$.
7. $6a + 5c$.
8. $z - 5$.
9. $-ab^2 - a^2b$.
10. $5x^2 + x + y$.
11. $x^2y - xz^2$.
12. $2ab + bd$.
13. $xy + 2yz + z^2$.
14. $2ab + 5bc - 2abc$.
1. $25\frac{2}{3}$.
2. 22.
3. 35.
4. 64.
5. 55.
6. $23\frac{1}{4}$.
7. $26\frac{1}{2}$.
8. $11\frac{1}{3}$.
9. 22.
10. $48\frac{1}{2}$.

Page 817.

1. $320.
2. 24.
3. 768.
1. $x = 12$.
2. $x = 30$.
3. $x = 60$.
4. $x = 48$.
5. $x = 18$.
6. $x = 60$.
7. 48.
8. 1040.

Page 818.

1. $x^3 + 2y - 1$.
2. $a^2 + 2b^2 - 2d^2 - 1$.
3. $x^2y - xy^2 + xy$.
4. $2xy - 2xy^2 + x^2z$.
5. $2ab^2 + 3ac^2$.
6. 32.
7. $7250.
8. $1350; $3600.
9. $40; $220.
10. $1200.
11. $7200.
12. 240.
13. 320.

Page 821.

1. $10a^3b - 2a^3b^2 + 6a^2b^2 - 8a^2bc + 14a^2b^3$.
2. $3b^4c^5 + 6b^3c^2 - 6b^2c^4 + 3b^5c^4 - 12b^2c^3$.
3. $8ac^4 + 8ac^5 - 4a^3c^4 + 12a^2c^3 - 4a^3c^5$.
4. $2x^3y^3 - 6x^2y^2 + 10xy^4 - 2x^4y^4 + 6xy^3$.
5. $3a^4b^3 + 3a^3b^4 - 15a^3b^2 + 9a^2b^3 - 12a^2b^4$.
6. $15a^3x^3 - 5a^4x^4 + 20a^4x^2 - 5a^5x^3 + 10a^3x^4$.
1. $21a^2 - 13ac - 20c^2$.
2. $20c^3 - 30cx^3 - 28c^2x + 42x^4$.
3. $18c^2 - 24cy - 10y^2$.
4. $15a^3 - 12ab^2 - 25a^2b + 20b^3$.
5. $32x^2 - 68xy + 21y^2$.
6. $20x^3 - 10xy^2 + 12x^2y - 6y^3$.
7. $18a^2 + 85ab + 18b^2$.
8. $14x^4 + 10xy^2 - 28x^3y - 20y^3$.

Page 822.

1. $12a^5 + 5a^3 - 24a^2 - 25a + 30$.
2. $5x^6 - 8x^5 + x^4 + 2x^3$.
3. $20x^4 - 22x^3 + 31x^2 - 15x$.

4. $3c^4 - 5c^3 + 9c^2 - 6c + 4$.
5. $12a^4 + 11a^3 + 6a^2 + 28a - 15$.
6. $15x^4 + 22x^3 - 12x^2 - 11x + 4$.
7. $3x^5y^2 - 4x^4y^3 - 8x^3y^4 + 3x^2y^5$.
8. $2a^4 + 3a^3b - 4a^2b^2 - 3ab^3 + 2b^4$.
9. $-6a^4 + 19a^3c - 3a^2c^2 - ac^3 + 3c^4$.
10. $6x^5 - 6x^4 - 7x^3 + 7x^2 + 14x - 15$.
11. $x^5 - x^4y - 2x^3y^2 + 2x^2y^3 + xy^4 - y^5$.
12. $-4m^5 + 12m^4 - 5m^3 + 2m^2 - 4m - 16$.
13. $2x^6 + 5x^5 + x^4 - 6x^3 - 9$.
14. $6a^6 - 17a^5 + 13a^4 - 12a^3 + 11a^2 - 9a - 4$.
15. $36x^5 - 15x^4y + 8x^3y^2 + 13x^2y^3 - 10xy^4 + 4y^5$.

Page 826.

1. 36.
2. 100.
3. 240.
4. 24.
5. 294.
6. $45.
7. 430.
8. 48 ft.
9. 24 rd.; 48 rd.
10. 130.
11. $725.40.
12. $575.
13. $760.

Page 828.

1. $3c - 4a + 5a^2c^2 - 2c$.
2. $2x^2 + xy - 4x + 1$.
3. $3bc + 5bc^2 - 4b^2 - b$.

Page 829.

4. $3ab - a^2b^2 + 5b - 2$.
5. $2xy + y - 4y^2 + 2x$.
6. $ac^2 + 2a^2 - ac + 1$.
7. $bc + 2c^2d - 4c + 3$.
8. $12ax + 3a^2x - 6y - 11$.
9. $2x^2 - xyz + 3x^2z - 4x$.
10. $3c + 2b - 4ac + 1$.
11. $4ab - 2b^2cd + 2bc^2 - b^2$.
12. $2ab + 4bcd - 3c^2 + 1$.
13. $4b^2y - 6bcx^2 + 5b^2x - 2$.
14. $3ac - 2bd + 5c - a$.

Page 830.

1. $a + 18$.
2. $x - 12$.
3. $x^3 + 8$.
4. $a^2 + 12$.
5. $y^2 - 16$.
6. $b^3 - 8$.
7. $a^3 + 48$.
8. $y^2 + 48$.
9. $a^3 + c^3$.
10. $b^2 + x^2$.
11. $a^2 + x^2$.
12. $a^2 - ac + c^2$.
13. $c^2 - b$.
14. $x^2 - 2xy + y^2$.
15. $x - y - z$.
16. $a - x + y$.
17. $4x - 1$.
18. $2a^3 - 3a - 6$.
19. $4b^2 - 3a^2$.
20. $8a^3 + 12a^2x - 18ax^2 - 27x^3$.
21. $5x^3 + 4x^2 + 3x + 2$.
22. $x^2 - x + 1$.
23. $16a^2 - 8ab + 4b^2$.
24. $a^4 - a^3 + a^2 - a + 1$.
25. $9x^2 - 6xy + 4y^2$.
26. $x^2 - 2x + 4$.
27. $6b + 9c$.
28. $4 - 2x + x^2$.
29. $5x - 4y$.
30. $a^3 + a^2b + ab^2 + b^3$.
31. $8a^3 + 12a^2b + 18ab^2 + 27b^3$.
32. $x^4 - x^3y + x^2y^2 - xy^3 + y^4$.
33. $27x^3 - 18x^2y + 12xy^2 - 8y^3$.
34. $x^4 - x^2y^2 + y^4$.
35. $4a + 8b$.
36. $a^6 + a^4y^2 + a^2y^4 + y^6$.
37. $7x - 5y$.

Page 831.

1. 130.
2. 320.
3. 216.
4. 112.
5. 1200.
6. 90.
7. 30.
8. 1296.
9. 2340.
10. 32.
11. 75.
12. 24.
13. 1800.
14. 300.
15. 4860.

Page 832.

1. 60.
2. 186.
3. $65.
4. 225.
5. 176.
6. 75.
7. $168.
8. 68.
9. 23.
10. $34.
11. $200.
12. 95.

Page 833.

1. $4x^4 + x^3 - 17x^2 + 7x + 6$.
2. $3a^4 - 17a^3 + 29a^2 - 22a + 6$.
3. $a^4 - 3ab^3 - 2b^4$.
4. $x^4 + x^3y - 12x^2y^2 + 2x^2 + 29xy - 15$.
5. $b^4 - 6b^3c + 8b^2c^2 + 2bc^3 - c^4$.
6. $a^8 + a^4b^4 + b^8$.
7. $3x^7 - 19x^5 + 2x^4 + 20x^3 - 16x^2 + 30$.
8. $60x^4 - 63x^3 + 104x^2 - 34x + 24$.

9. $15x^3 + 66x^3y^2 - 16x^2y + 88x^2y^3 - 48xy^4$.
10. $16a^5 + 20a^3 + 9a$.
11. $-x^5 + x^4 + 9x^3 - x^2 + x + 1$.

1. $a^2 - 3a - c$.
2. $x^2 + 2xy + 3y^2$.
3. $b^2 - 2bc + 4c^2$.
4. $2a + 3c$.
5. $5b - 3c$.
6. $2x^2 - 3x + 1$.

7. $9y^2 - 9y - 4$.
8. $a - 3$.
9. $x^3 + 3x^2y + 3xy^2 + y^3$.
10. $x + y$.
11. $2a^3 - a + 1$.
12. $27x^2 + 12xy + 6y^2 - 1$.

Page 834.

1. 144.
2. 40.
3. $160; $400.
4. $165.
5. $42.
6. 48.
7. $7000.
8. $135.
9. All, $24.
10. $180.

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